SYLLABUS FOR M.A./M.SC.INMATHEMATICS

Under Choice Based Credit System (CBCS)

Effective from 2020-2021

The University of Burdwan
Burdwan-713104
West Bengal

Preamble

The M.A./M.Sc. in Mathematics is a two-year, four-semester programme under Choice Based Credit System (CBCS). Total 96 credits are equally distributed in four semesters (24 credits in each semester). 74, 16, 4, 2 and 2 credits are given for 18 core courses, 4 discipline-centric Elective courses, project, 1 Generic elective course and Community Engagement Activities respectively. The programme is divided into two streams, namely Applied and Pure after 2ndsemester. Discipline-centric Elective courses can be chosen from the baskets which contain a variety of courses. The courses in 1st and 2nd semester are designed on considering the syllabi of various eligibility tests at the national level.

Objectives

- Impart teaching so that the students could develop critical thinking ability about the fundamental aspects of mathematics.
- Make the students capable of pursuing research work in various emerging fields of mathematics and its applications.
- Train the students with mathematical knowledge and computational techniques so that they can deal with the problems faced by the industries.
- Make the students aware of their responsibility to meet the societal needs.

Pre-requisite

The students should possess the knowledge on the courses taught in the B.A./B.Sc. with Mathematics Honours.

Programme Outcomes

- Development of critical thinking for carrying out scientific investigations.
- Skills to analyze problems, formulate a hypothesis, evaluate and validate results, and draw reasonable conclusions thereof.
- Ability to pursue research or build careers in industry in mathematical sciences and allied fields.

- Development of the effective scientific and technical communications in both oral and written forms.
- Awareness for becoming a responsible citizen with commitment to deliver one's responsibilities within the scope of bestowed rights and privileges.

Programme Specific Outcomes

- Understanding about the fundamental axioms in mathematics and capability of developing ideas based on them.
- Development of mathematical reasoning and an understanding of the underlying unifying structures of mathematics (i.e., sets, relations and functions, logical structure) and the relationships among them.
- Motivation for research studies in mathematics and related fields with real life applications.
- Knowledge in a wide range of mathematical techniques and application of mathematical methods/tools in other scientific and engineering domains.
- Advanced knowledge in mathematical sciences.
- Nurturing problem solving skills, thinking, creativity through assignments, project work.
- Preparing for competitive examinations, like NET, SET, GATE, etc.

PROFILE

Semester I

Course				Lect. Hr /week	Dur. of Exam (in H)	Marks			Credit
Course code	Type	T/P	Name			I.A.	E.T	Total	
MSMG101	Core	T	Real Analysis	4	2	10	40	50	4
MSMG102	Core	T	Complex Analysis	4	2	10	40	50	4
MSMG103	Core	T	Foundation of Topology	4	2	10	40	50	4
MSMG104	Core	T	Integral Transforms and Integral Equations	4	2	10	40	50	4
MSMG105	Core	T	Classical Mechanics	4	2	10	40	50	4
MSMG106	Core	T	Numerical Analysis	4	2	10	40	50	4
					Total credit				24

Abbreviation used: T/P → Theory/Practical; I.A.→ Internal Assessment; E.T. → Term-end examination

Semester II

Course				Lect. Hr /week	Dur. of Exam (in H)	Marks			Credit
Course code	Type	T/P	Name			I.A.	E.T	Total	
MSMG201	Core	T	Algebra	4	2	10	40	50	4
MSMG202	Core	T	Elements of Functional Analysis and Multivariate Calculus	4	2	10	40	50	4
MSMG203	Core	T	Geometry of Curves and Surfaces	4	2	10	40	50	4
MSMG204	Core	T	Differential Equations	4	2	10	40	50	4
MSMG205	Core	T	Operations Research	4	2	10	40	50	4
MSMG206	Core	P	Computer-aided Numerical practical	8	4	10	40	50	4
					Total credit				24

Semester III (Applied Stream)

Course				Lect. Hr /week	Dur. of Exam (in H)	Marks			Credit
Course code	Type	T/P	Name			I.A.	E.T	Total	
MSMA301	Core	T	Methods of Applied Mathematics	4	2	10	40	50	4
MSMA302	Core	T	Continuum Mechanics	4	2	10	40	50	4
MSMA303	Core	T	Theory of Electromagnetic fields and Special Theory of Relativity	4	2	10	40	50	4
MSMA304	GE	T	*	2	1	5	20	25	2
MSMA305	DE	T	**Vide Appendix I**	4	2	10	40	50	4
MSMA306	DE	T	**Vide Appendix II**	4	2	10	40	50	4
MSMA307	CE	N.A.	N.A.	N.A.	N.A.	5	20	25	2
					Total credit				24

Abbreviation used: CE → Community Engagement Activities; DE → Discipline-centric Elective (One Discipline-centric Elective course in Semester III may be opted from SWAYAM); GE → Generic elective (Generic elective course may be opted from SWAYAM).

*: MSMA304-1: Introduction to Operations Research; MSMA304; MSWM304: Course opted from SWAYAM.

Appendix I

Basket of courses for DE (Only one course has to be chosen from the basket)

MSMA305-1: Boundary Layer Theory and Magneto-hydrodynamics-I
MSMA305-2: Turbulent Flows-I
MSMA305-3: Space Sciences-I
MSMA305-4: Course opted from SWAYAM

Appendix II:

Basket of courses for DE (Only one course has to be chosen from the basket)

MSMA306-1: Advanced Optimization-I
MSMA306-2: Advanced Operations Research-I
MSMA306-3: Quantum Mechanics-I
MSMA306-4: Fuzzy Mathematics and Applications-I
MSMA306-5: Course opted from SWAYAM

Semester III (Pure Stream)

Course				Lect. Hr /week	Dur. of Exam (in H)	Marks			Credit
Course code	Type	T/P	Name			I.A.	E.T	Total	
MSMP301	Core	T	Abstract Algebra	4	2	10	40	50	4
MSMP302	Core	T	Analysis I	4	2	10	40	50	4
MSMP303	Core	T	Geometry of Manifolds	4	2	10	40	50	4
MSMP304	GE	T	*	2	1	5	20	25	2
MSMP305	DE	T	**Vide Appendix III**	4	2	10	40	50	4
MSMP306	DE	T	**Vide Appendix IV**	4	2	10	40	50	4
MSMP307	CE	N.A.	N.A.	N.A.	N.A.	5	20	25	2
					Total credit				24

Abbreviation used: CE → Community Engagement Activities; DE → Discipline-centric Elective (One Discipline-centric Elective course in Semester III may be opted from SWAYAM); GE → Generic elective (Generic elective course may be opted from SWAYAM).

*: MSMP304-1: Introduction to Graph Theory, MSWM304: Course opted from SWAYAM.

Appendix III

Basket of courses for DE (Only one course has to be chosen from the basket)

MSMP305-1: Advanced Functional Analysis-I
MSMP305-2: Advanced Differential Geometry – I
MSMP305-3: Advanced Complex Analysis-I
MSMP305-4: Measure and Integration-I
MSMP305-5: Course opted from SWAYAM

Appendix IV

Basket of courses for DE (Only one course has to be chosen from the basket)

MSMP306-1: Euclidean and non – Euclidean Geometries-I
MSMP306-2: Commutative Algebra-I
MSMP306-3: Advanced Operator Theory –I
MSMP306-4: Ergodic Theory-I
MSMP306-5: Algebraic Topology-I
MSMP306-6: Course opted from SWAYAM

Semester IV (Applied Stream):

Course				Lect. Hr /week	Dur. of Exam (in H)	Marks			Credit
Course code	Type	T/P	Name			I.A.	E.T	Total	
MSMA401	Core	T	Dynamical Systems, Chaos and Fractals	4	2T/4P	10	40	50	4
MSMA402	Core	T	Fluid Mechanics	4	2T/4P	10	40	50	4
MSMA403	Core	T	Introduction to Quantum Mechanics and Wavelet Analysis	4	2T/4H	10	40	50	4
MSMA404	DE	T	**Vide Appendix V**	4	2T/4H	10	40	50	4
MSMA405	DE	T	**Vide Appendix VI**	4	2T/4H	10	40	50	4
MSMA406	Project	N.A.	N.A.	4	---	---	50	50	4
					Total credit				24

Appendix V

Basket of courses for DE (Only one course has to be chosen from the basket)

MSMA404-1: Boundary Layer Theory and Magneto-hydrodynamics-II
MSMA404-2: Turbulent Flows-II
MSMA404-3: Space Sciences-II

Appendix VI

Basket of courses for DE (Only one course has to be chosen from the basket)

MSMA405-1: Advanced Optimization-II
MSMA405-2: Advanced Operations Research-II
MSMA405-3: Quantum Mechanics-II
MSMA405-4: Fuzzy Mathematics and Applications-II

Semester IV (Pure Stream):

Course				Lect. Hr /week	Dur. of Exam (in H)	Marks			Credit
Course code	Type	T/P	Name			I.A.	E.T	Total	
MSMP401	Core	T	Graph Theory, Set Theory and Logic	4	2T/4P	10	40	50	4
MSMP402	Core	T	Analysis II	4	2T/4P	10	40	50	4
MSMP403	Core	T	Topology	4	2T/4H	10	40	50	4
MSMP404	DE	T	**Vide Appendix VII**	4	2T/4H	10	40	50	4
MSMP405	DE	T	**Vide Appendix VIII**	4	2T/4H	10	40	50	4
MSMP406	Project	N.A.	N.A.	4	---	---	50	50	4
					Total credit				24

Appendix VII

Basket of courses for DE (Only one course has to be chosen from the basket)

MSMP404-1: Advanced Functional Analysis-II
MSMP404-2: Advanced Differential Geometry – II
MSMP404-3: Advanced Complex Analysis-II
MSMP404-4: Measure and Integration-II

Appendix VIII

Basket of courses for DE (Only one course has to be chosen from the basket)

MSMP405-1: Euclidean and non – Euclidean Geometries-II
MSMP405-2: Commutative Algebra-II
MSMP405-3: Advanced Operator Theory –II
MSMP405-4: Ergodic Theory-II
MSMP405-5: Algebraic Topology-II

DETAILED SYLLABUS

Course: MSMG101
Real Analysis (Marks - 50)

Total lectures Hours: 50H

Objectives

To present the concepts of a function of bounded variation, Riemann-Stieltjes integral, Lebesgue measure, Lebesgue integral and Fourier series.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. functions of bounded variation and its characterization.
ii. Riemann-Stieltjes integral as a generalization of Riemann integral.
iii. Lebesgue measure, measurable sets, measurable functions, Lebesgue integral and its relationship with Riemann integral.
iv. Fourier series and its basic properties.

Skills: The students would be able to

i. characterize functions which are of bounded variation.
ii. find Lebesgue measure of various subsets of $\mathbb{R}$.
iii. determine functions which are Lebesgue measurable.
iv. calculate Lebesgue integral of measurable functions.
v. calculate Fourier series of suitable functions.

General competence: The students would gain

i. general idea of Riemann-Stieltjes integral, Lebesgue measure, Lebesgue integral and Fourier series which will be useful for further studies in real analysis, probability theory.
ii. analytical and reasoning skills, which improve their thinking power.

Contents:

Monotone functions and their discontinuities, functions of bounded variation and their properties, characterization of a function of bounded variation. [5H]

Riemann-Stieltjes integral, necessary and sufficient condition for existence of Riemann-Stieltjes integral, integration by parts, change of variables in integral, integral of step functions, first mean value theorem and second mean value theorem for Riemann-Stieltjes integrals. [5H]

Lebesgue outer measure, properties satisfied by Lebesgue outer measure, measurable sets and their properties, non-measurable sets, Lebesgue measure, notions of σ-algebra, Borel sets, F_σ–sets, G_δ –sets. [10H]

Measurable functions, continuity and measurability, monotonicity and measurability, measurability of supremum and infimum, simple functions, sequence of measurable functions, Egorov's theorem. [5H]

The Lebesgue Integral: Lebesgue integral of a nonnegative measurable function (bounded or unbounded), integrable functions and their simple properties, Lebesgue integral of functions of arbitrary sign, integral of pointwise limit of sequence of measurable functions, Lebesgue monotone convergence theorem and its consequences, Fatou's lemma, dominated convergence theorem, comparison of Lebesgue and Riemann integral, Lebesgue criterion of Riemann integrability. [15H]

Differentiation and Integration: Vitali covering lemma, differentiation of monotone functions, differentiation of an integral, absolute continuity, fundamental theorem of integral calculus for Lebesgue integral. [5H]

Fourier series, Dirichlet's kernel, Riemann Lebesgue theorem, pointwise convergence of Fourier series of functions of bounded variation. [5H]

Text Books:

1. Measure Theory and Integration. G. D. Barra (New Age International (P) Ltd, 2013).
2. Real Analysis. H.L.Royden, 3rdedition (Pearson, 1988).
3. Mathematical Analysis.T. M. Apostol (Narosa, 1985).

Reference Books:

1. Measure Theory. P. R. Halmos (Springer-Verlag, 1974).
2. Measure and Integration. S. K. Berberian,(The Macmillan Co., 1965)
3. Real Analysis. B.K.Lahiri and K.C.Roy (World Press, 1988).
4. A second Course of Mathematical Analysis.J. C. Burkil & H. Burkil (CUP, 1980).
5. Real Analysis.R. R. Goldberg (Springer-Verlag, 1964).
6. Theory of Functions of a Real Variable, Vol. I. I. P. Natanson (Fedrick Unger Publi. Co., 1961).
7. Principle of Mathematical Analysis. W. Rudin (Mc Graw Hill, 1964).
8. Measure, Integration and Function Spaces.Charles Swartz (World Scientific, 1994).
9. Theory of Functions.E. C. Titchmarsh (CUP, 1980).

Course: MSMG102
Complex Analysis (Marks - 50)

Total lectures Hours: 50H

Objectives

To present a systematic introduction of the fundamental course on complex analysis.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. basic concepts of analytic functions, entire functions, complex integration.
ii. comparative study between Frechet differentiability and complex differentiability via Cauchy Riemann equation.
iii. homotopy version of Cauchy's theorem.
iv. the ideas on singularities with their classifications and its role on some important theorems like Rouche's theorem, Casorati-Weierstrass theorem etc.
v. Mobius transformations, Riemann Mapping theorem and analytic continuation with its geometric interpretations.

Skills: The students would be able to

i. check analyticity of a complex valued functions
ii. integrate a complex valued function having certain types of singularities.
iii. learn a geometrical study of Mobius transformations.

General competence: The students would gain their

i. overall expertise in dealing with complex integration and computing residues.
ii. expertise in solving many applied problems using complex analysis.

Contents:

(Prerequisite: Basic properties of analytic functions, entire functions, complex integrations, Cauchy's theorem and integral formula). [4H]
Homotopy version of Cauchy's theorem, simply connected region and primitives of analytic functions, Morera's theorem. [4H]
Zeros of an analytic function, singularities and their classifications, Riemann's theorem, limit points of zeros and poles, Casorati-Weierstrass's theorem, behaviour of a function at the point at infinity, winding number, counting zeros, open mapping theorem. [12H]
Theory of residues, Cauchy's residue theorem and evaluation of improper integrals, Argument principle, Rouche's theorem and its applications, maximum modulus theorem, Schwarz's lemma. [14H]
Conformal mappings, Mӧbius transformations, Riemann mapping theorem. [10H]
Introduction to Analytic continuation, Monodromy theorem. [6H]

Text Book:

1. Functions of one Complex variable.J. B. Conway, 2nd edition (Narosa Publishing House, New Delhi, 1997).

Reference Books:

1. Complex Analysis.K. Kodaria (Cambridge University Press, 2007)
2. Theory of Complex functions.R. Remmmert (Springer-Verlag, New York, 1991).

3. Complex Analysis.L. V. Ahlfors,3rd edition (McGraw-Hill, 1979).
4. Complex Variables and applications.R. V. Churchill and J. W. Brown (McGraw-Hill, 1996).
5. Theory of Functions of a Complex Variable (Vol. I, II & III). A. I. Markushevich (Prentice-Hall (1965& 1967).
6. The Theory of Functions.E. C. Titchmarsh (Oxford University Press,1939).
7. Introduction to the Theory of Function of a Complex Variable.E. T. Copson (Oxford University press, 1970).
8. Elementary Theory of Analytic Functions of One or Several Complex Variables.H. Cartan (Dover Publication, 1995).
9. Real and Complex Analysis.W. Rudin (Tata Mc Graw-Hill Education, 1987).
10. Foundation of Complex Analysis. S Ponnuswamy, 2ndedition (Narosa Publishing House, 2018).

Course: MSMG103
Foundation of Topology (Marks - 50)

Total lectures Hours: 50H

Objectives

To present a systematic introduction of the fundamentals course on topology.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. generation of topological spaces via different methods.
ii. formations of new topology using given topological spaces like products, sums, identifications etc.
iii. concepts of continuity, homeomorphisms, open and closed maps etc.
iv. the ideas on countability axioms, separation axioms
v. structural similarities and dissimilarities of compactness, Lindeloffness and connectedness

Skills: The students would be able to

i. find the existence of a topology on a set via several techniques.
ii. construct counter examples using products, sum, quotient etc.

iii. distinguish and set examples and counter examples using different types of topological spaces like first and second countable spaces, separable spaces, $T_0, T_1, T_2, T_3, T_4\, T_{3\frac{1}{2}}$ etc.

iv. compare systematically Lindeloff, compact, locally compact and connected spaces.

General competence: The students would gain

i. general idea of constructing examples and counter examples in topology

ii. understanding in fundamental concepts in compactness, local compactness and connectedness.

iii. experience to understanding proofs of few fundamental theorems in topology like Urysohn's lemma, Tietze Extension theorem, Alexander subbase theorem, Tychonoff product theorem.

Contents:

Topological spaces: Open sets, closed sets, closure, denseness, neighbourhoods, interior points, limit points, derived sets, bases, subbases, subspaces, generation of a topology using Kuratowski closure operator and neighbourhood systems, continuous functions, pasting lemma, open maps and closed maps, homeomorphism and topological invariants, sum and product of arbitrary many topological spaces**.** [12H]

Separation axioms: $T_0, T_1, T_2, T_3, T_4, T_5$spaces, their properties, characterizations and their relationship. Regularity, complete regularity, $T_{3\frac{1}{2}}$-spaces, normality, complete normality and their characterizations and basic properties. Urysohn's lemma, Tietze's extension theorem. [12H]

Countability axioms: First and second countable spaces, Lindelöf spaces, separable spaces. Properties on continuity and subspaces of first and second countable spaces, Lindelöf spaces, separable spaces. [6H]

Compactness: Compact spaces, compact subspaces, characterizations in terms of finite intersection property, Alexander subbase theorem, Tychonoff product theorem, compactness and separation axioms, compactness and continuous functions, sequentially, Fr$\acute{e}$chet and countably compact spaces, subspaces and their mutual relationship, locally compact spaces. [12H]

Connectedness: connected spaces and their characterizations, connected subspaces, connectedness of the real line, components, totally disconnected spaces, locally connected spaces, path connectedness, path components, locally path connected spaces**.** [8H]

Text books:

1. Topology. J. R. Munkres, 2nd edition (Pearson, 2015)
2. Introduction to General Topology. K. D. Joshi, 2nd edition (New Age International, 2018).

3. Introduction to Topology and Modern Analysis. G. F. Simmons, 1st edition (Tata McGraw-Hill, 2015).

Reference books:

1. General Topology. S. Willard (Dover Publication,INC, 2004).
2. Topology. K. Janich, 1st edition (Springer Verlag, 2006).
3. Encyclopedia of General Topology. P.K.Hart,J. Nagata, J.E. Vaughan, 1st edition (Elsevier,2003)
4. Introduction to General Topology. S.T. Hu, 1st edition (Holden Day, 1966)
5. Foundation of Topology, C.W Patty, 2nd edition (Jones and Bartlett Publishers, Inc, 2008)
6. Topology, J. Dugundji, 1st edition (Mc-Graw Hill Inc.,US, 1988)
7. General Topology (Chapter 1-4), N. Bourbaki, 1st edition (Springer, 1998)
8. Topology, J. Dugundji, 1st edition (Ubs Publishers Distributors Ltd, 1999)
9. Counter examples in topology,L.A. Steen, J.A.Seebach, 2nd edition (Dover Publication,Inc.,1995)
10. Introduction to General Topology,J.L. Kelly, 1st edition (Springer India, 1988).
11. General Topology, R. Engelking, 2nd edition (Helderman Verlag, 1989).

Course: MSMG104
Integral Transforms and Integral Equations(Marks - 50)

Total lectures Hours: 50H

Objectives

To present the theories for studying integral equations and integral transforms in a systematic manner.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. mathematical foundations of integral equations and integral transforms including techniques of solving those

Skills: The students would be able to

i. construct integral equations from physical problems and to convert it to a differential equation and vice-versa

ii. gain techniques for solving integral equations and evaluating integral transforms

General competence: The students would gain

i. general idea about the integral equations and integral transforms
ii. understanding about fundamental solution procedures of integral equations

Group A

Integral Transforms (Marks-25)

Contents:

Introduction: Definition of Integral Transform, Kernel of a transform, Derivation of different integral transforms. [3H]

Laplace transform: Definition and its existence, Basic properties of Laplace transform, Laplace transform of derivatives and its asymptotic properties (Initial value and Final value Theorem), Laplace transform of an integral, Inversion by analytical method and Bromwich path, Convolution theorem, Heaviside's series expansion, Applications to ordinary and partial differential equations. [11H]

Fourier transform: Definition and basic properties, Fourier transform of some elementary functions, Fourier transform of derivatives, Inverseformula, Convolution theorem, Parseval's relation, Fourier sine and cosine transforms, Finite Fourier Transform. Applications to Heat, Wave and Laplace equations. [11H]

Group B

Integral Equations (Marks-25)

Contents:

Linear Integral equation, classification, conversion of initial and boundary value problems to an integral equation and vice-versa, Different types of kernels, Eigen-Values and Eigen functions. [4H]

Solutions of homogeneous and general Fredholm integral equations of second kind with separable kernels. [4H]

Existence, Uniqueness and iterative solution of Fredholm and Volterra Integral equations; Resolvent kernel, Solution of Volterra integral equation of first kind, Integral equations of Convolution type and their solutions by Laplace transform. [6H]

Integral equations with symmetric kernels: Orthogonal system of functions, fundamental properties of eigen values and eigen functions for symmetric kernels. [6H]

Fredholm theorems. Solution of Volterra integral equations with convolution type kernels. [3H]

Singular integral equation, Solution of Abel's integral equation. [2H]

Text Books:

1. The use of Integral Transforms. Ian N. Sneddon, 2nd edition(McGraw Hill,1972).
2. Fourier Transforms. Ian N. Sneddon, 2nd edition (Dover Publications,2010).
3. Linear Integral Equations. W. V. Lovitte (Dover Publications, 2005).

4. Linear Integral Equations. R. P. Kanwal, 2nd edition (Birkhäuser, 1996).

Reference Book:

1. Linear Integral Equations. S.G. Mikhlin, 1st edition (Routledge, 1961).
2. Integral Equation. F. G. Tricomi (Dover Publications, 2012).
3. Integral Transforms and their applications. Loknath Debnath, 2nd edition (Chapman and Hall/CRC,2006).
4. Schaum's Outline of Laplace Transforms (Schaum's Outlines of Theory and Problems), Murray Spiegel, 1st edition (McGraw-Hill Education, 1965).

Course: MSMG105
Classical Mechanics (Marks - 50)

Total lectures Hours: 50H

Objectives

To impart knowledge and understanding of fundamental concepts of dynamics of system of particles, motion of a rigid body, constrained motion, Lagrangian and Hamiltonian formalism, canonical transformations and brackets.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. mathematical formalisms of classical mechanics.
ii. conservation laws, symmetries, angular momentum and spin.
iii. canonical transformations, generating functions and brackets.
iv. functional, Euler-Lagrange equations.

Skills: The students would be able to

i. describe the motion of a mechanical system using Lagrangian and Hamiltonian formalism.
ii. describe motion of a rigid body.
iii. solve some physical systems moving under some constraints.
iv. use Euler-Lagrange equations to find stationary paths.
v. evaluate Lagrange and Poisson brackets.
vi. develop Hamilton-Jacobi method to solve equations of motion.

General competence: The students would gain

i. general idea of classical mechanics which will be useful for further studies in theoretical physics.
ii. understanding about fundamental mechanical processes in nature.
iii. experience to construct approximate mechanical models using mathematical tools.

Contents:

Mechanics of a particle and system of particles: Mechanics of a particle, Mechanics of a system of particles, Conservation of linear momentum, angular momentum and energy, Constraints, Classification of constraints and their examples, Degrees of freedom, Generalised coordinates, Limitations of Newton's laws of motion. [6H]

Motion of a rigid body: Rotating coordinate system, Two-dimensional motion of a rigid body rotating about a fixed point, Euler's dynamical equations and solution, Invariable line and invariable plane, Torque free motion, Euler angles, Components of angular velocity in terms of Euler angles, Motion of a top in a perfectly rough floor, Stability of top motion.[8H]

Lagrangian mechanics: Lagrange's equation of motion of the first kind, Gibbs-Appell's principle of least constraint, Lagrange's equations of motion of the second kind (holonomic and non-holonomic systems), Velocity dependent potential, Dissipative forces, Rayleigh's dissipation function, Generalised momenta and energy, Gauge function for Lagrangian, Cyclic coordinates, Routh process for ignorable coordinates, Noether's theorem, Symmetry and conservation laws. [10H]

Hamiltonian mechanics: Legendre dual transformation, Hamilton's canonical equations of motion, Hamilton's principle, Derivation of Hamilton's equations of motion, Invariance of Hamilton's principle under coordinate transformation, Principle of least action. [8H]

Calculus of variations: Derivation of Euler-Lagrange's equation of motion, Sufficient condition for existence of extremals, Brachistochrone problem, Geodesic, Isoperimetric problem, Variational problems with moving boundaries. [8H]

Canonical transformations: Definition, examples and properties of canonical transformations, Generating functions, Lagrangian and Poisson brackets (definition and properties), Poisson's theorems, Condition of canonicality in terms of Poisson bracket, Infinitesimal canonical transformations, Hamilton-Jacobi's equation. [10H]

Text Books:

1. Classical Mechanics.H. Goldstein (Narosa Publ. House, 1997).
2. Classical Mechanics. N. C. Rana & P.S. Jog (Tata McGraw Hill, 2001).

Reference Books:

1. Classical Mechanics with Introduction to Nonlinear Oscillation and Chaos. V. B. Bhatia (Narosa Publishing House, 1997).
2. A Treatise on the Analytical Dynamics of Particles and Rigid Bodies. E. T. Whittaker (Cambridge University Press, 1993).
3. Calculus of Variations. I. M. Gelfand and S.V. Fomin (Prentice Hall Inc., 2012).
4. Calculus of Variations with Applications. A. S. Gupta (Prentice–Hall of India, 1996).

Course: MSMG106
Numerical Analysis (Marks - 50)

Total lectures Hours: 50H

Objectives

To study and analyse numerical methods for solving problems

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. solution of system of linear and non-linear equations by different numerical methods.
ii. error analysis of the methods.
iii. solution of ordinary and partial differential equations.

Skills: The students would be able to

i. apply the numerical techniques for practical problems.
ii. perform error analysis of the methods.

General competence: The students would gain

i. general idea of computation techniques and algorithms.
ii. implementation of numerical methods to the programming language

Contents:

Numerical solutions of non-linear equations: Modified Newton-Raphson method, Aitken δ^2 method, Newton and Quasi-Newton methods for non-linear system of equations.
Roots of polynomial equations: Bairstow method, Graeffe's root squaring method and their convergences. [6H]
Solution of linear systems: Partial pivoting, Complete pivoting, Operation counts, Triangular Factorization methods, Matrix Inversion method, Successive-Over Relaxation (SOR) iteration method, Convergence, Concept of ill condition systems. [6H]
Polynomial approximations: Existence and uniqueness of polynomial interpolations, Piecewise polynomial interpolation, Hermite interpolation, Error analysis, Cubic spline interpolation, Convergence properties, Least square polynomial approximations, Approximations using orthogonal polynomials, Chebyshev polynomials. [7H]
Numerical integration: Gauss-Legendre and Gauss-Chebyshev quadratures, Euler-Maclaurin summation formula, Richardson extrapolation, Romberg integration, Improper integrals. [6H]

Eigenvalues and eigenvectors of a matrix: Power method for largest eigenvalue and corresponding eigenvector, Inverse power method, Jacobi method and convergences. [6H]
Solutions of Initial Value Problems: Solution of first order ordinary differential equation by multi-step predictor-corrector method, Adams-Bashforth method, Adams-Moulton method and Milne's method, Runge-Kutta method for second order ordinary differential equation, Runge-Kutta time-steped technique, Convergence and stability. [6H]
Solutions of Boundary Value Problems: Finite difference method, Shooting method for the solution of linear and non-linear equations, Introduction to finite element method. [6H]
Numerical solution of partial differential equations by finite difference method: Explicit and Implicit methods, Consistency, Convergence and stability, Lax theorem, Heat equation: Explicit and Implicit Crank-Nickolson methods, Wave equation: Explicit finite difference method, Stability analysis. [7H]

Text Books:

1. An Introduction to Numerical Analysis. K. E. Atkinson (John Wiely& Sons, Singapore, 1989).
2. Elementary Numerical Analysis: An Algorithmic Approach. S.D. Conte and C. De Boor (Mc Graw Hill, New York, 1980).
3. Introductory Methods of Numerical Analysis. S. S. Sastry (PHI, New Delhi, 1999).

Reference Books:

1. A First Course in Numerical Analysis. A. Ralston (McGraw Hill, New York, 1965).
2. Numerical Methods for Scientific and Engineering Computation. M. K. Jain, S. R. K. Iyenger& P. K. Jain, 4th edition (New Age International (P) Ltd., New Delhi, 2003).
3. Numerical Solution of Partial Differential Equations. G.D. Smith (Oxford University Press, 1985).

Course – MSMG 201
Algebra (Marks - 50)

Total lectures Hours: 50H

Objectives

To present a systematic introduction of the fundamentals course on algebra.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. action of a group on a set and its various connection with some important concepts in group theory like Sylow's theorems.
ii. application of Sylow's theorem in representation of all finite abelian groups.

iii. application of Euclidean domain (ED), Unique Factorization domain (UFD) to prove Euclidean algorithm and Fundamental Theorem of Arithmetic in more general set up.
iv. procedure to find out Grobner basis for an ideal of polynomial ring of several variables.
v. application of Jordan Canonical form, complexification of vector space etc.

Skills: The students would be able to

i. Compute many problems in different field of mathematics using the action of groups like $SL_n(\mathbb{Z}), GL_n(\mathbb{Z})\ etc$ on various surfaces.
ii. Develop skills to check whether a group is simple or not using Sylow and to completely describe all abelian groups of any order.
iii. establish whether a polynomial over a field can be factored non-trivially and to provide examples and counter examples.
iv. manipulate Jordan Canonical form which is a very useful concept in other branches of mathematics.

General competence: The students would gain

i. general idea of constructing various groups upto isomorphism
ii. understanding in fundament concepts in Group action, Sylow, ED, PID, UFD, irreducibility of polynomials, complexification of vector spaces and its application.
iii. expertized in solving many tricky problems in algebra

Contents:

Group Theory: Properties of general Dihedral group $\boldsymbol{D_n}$ and Quaternion group of order 8, classification of groups of order 6 and non-commutative groups of order 8,Group actions, orbit of a point, stabilizer of a point, class equation and its importance, Cayley's theorem, and related results to the existence of normal subgroups, properties of p-groups, where p is prime. Sylow–p-subgroups and its properties, Sylows theorem I, II & III, classification of simple groups up to order 60, external and internal direct product of groups - their equivalencies, representation of finite abelian groups. [12H]

Ring Theory: Prime and irreducible elements in an integral domain, their relations in principal ideal domain (PID) and unique factorization domain (UFD), associates, gcd of two or more elements and their existence, importance of Euclidean domain (ED) in respect of computing gcd's, equivalence of PID and existence of Dedekind-Hasse norm, universal side divisor and its connection with PID, polynomial rings over a field Characterization of field in terms of ED and PID, polynomial rings that are not UFD, Gauss's lemma, R is UFD if and only if R[x] is UFD, examples and counter-examples related to ED, PID and UFD, polynomial rings of several variables over a field F, Hilbert basis theorem, monomial ordering, Gr$\boldsymbol{\ddot{o}}$bner basis for an ideal in $\boldsymbol{F[x_1, x_2, \dots, x_n]}$, general polynomial division, Buchberger's algorithm, solving system of algebraic equations by Gr$\boldsymbol{\ddot{o}}$bner bases. [13H]

Linear Algebra: Prerequisite (Similar and congruent matrices, characteristic polynomial, minimal polynomial, diagonalization, Cayley-Hamilton theorem), Jordan canonical form, bilinear forms, symmetric and skew symmetric bilinear forms, groups preserving bilinear forms (orthogonal, pseudo-orthogonal and Lorentz groups), matrix associated with a bilinear form, multilinear forms (definition and examples), quadratic form, rank, signature and index of a quadratic form, reduction of a quadratic form to its normal form, Sylvester's law of inertia, simultaneous reduction of two quadratic forms, applications to geometry**.** [15H]
Complexification of a vector space, complexification of an operator, minimal polynomial of the complexification, eigenvalues of the complexification, characteristic polynomial of the complexification, normal operators on real inner product spaces, isometries on the real inner product spaces, characterization of normal operators over real field, trace of an operator.
[10H]

Text books:

1. Abstract Algebra. D.S. Dummit & R.M. Foote, 3rd edition (Wiley, 2016).
2. Basic Abstract Algebra. P. B. Bhattacharya, S. K. Jain& S. R. Nagpaul, 2nd edition (Cambridge University Press, 2014).
3. Linear Algebra done right. Sheldon Axler, 3rd edition (Springer, 2015).
4. Linear Algebra. S. H. Friedberg, A.J. Insel &L.E. Spence, 4th edition (Prentice Hall of India pvt., 2004).

Reference books:

1. Algebra.T. W. Hungerford , 3rd edition (Cengage Learning India Pvt Ltd,2012).
2. A First Course in Abstract Algebra. J.B. Fraleigh, 7thedition (Pearson Education (India) 2002).
3. Abstract Algebra. M. Artin, 2ndedition (Pearson Education, 2011).
4. Contemporary Abstract Algebra. J. A. Gallian, 4th edition (Cengage Learning India Pvt Ltd 2019).
5. An Introduction to the Theory of Groups. J. Rotman, 4thedition (Springer 1999).
6. Topics in Algebra. I.N. Herstein, 2nd edition (John Wiley & Sons Inc (Sea) Pte Ltd, 2017).
7. Algebra. S. Lang, 3rd edition (Springer 2002).
8. Linear Algebra. S. K. Berberian (Oxford University Press, 1992).
9. Linear Algebra. S. Lang,3rd edition (Springer, 1987).
10. Fundamental of Abstract Algebra. D.S. Malik, J.N. Mordenson, M.K.Sen, 1st edition (Mcgraw-Hill Book Company – Koga, 1997).
11. Linear Algebra. K. Hoffman, R. Kunze, 2nd edition (Pearson Education Limited, 2016).

Course – MSMG202

Elements of Functional Analysis and Multivariate Calculus (Marks - 50)

Total lectures Hours: 50H

Group A

Elements of Functional Analysis (Marks -25)

Objectives

To familiarize the students with a systematic foundation of the knowledge on the fundamental results of functional analysis by which the knowledge of these results could be applied to analytic problems of applied mathematics or mathematical physics.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge :

The students acquire their knowledge:

- on metric fixed point theory particularly on Banach Contraction Principle.
- on some applications of Banach Contraction Principle
- the elementary theory on Banach spaces and Hilbert spaces together with the notions of :
  - (i) their interrelations
  - (ii) their geometrical study
  - (iii) the criterion for dimensionality of spaces vis-a vis compactness of a closed set
  - (iv) convergence of series in Banach Spaces
  - (v) some important inequalities namely Cauchy Schwartz Inequality , Bessel's inequality, Perseval's equality with their applications
  - (vi) the applications of orthogonal and orthonormal vectors in Hilbert spaces
  - (vii) square integrable functions and its study on its completeness property

Skills:

- The students are habituated and motivated in working out various problems independently on the allied topics.
- The students are influenced in searching for the close connection between the real analysis and topology up to a certain extent that enable them to solve the problems in different directions.

General Competence:

(i) It helps the students to read and to learn evaluate critically further topics in functional analysis

It motivates the students to make them easier in understanding the use of functional analysis in applied problems.

Contents:

Prerequisite on metric spaces, Baire's category theorem, applications of Banach contraction principle to (i) solutions of system of linear algebraic equations, (ii) implicit function theorem and (iii) Fredohlm integral equations. [6H]

Normed linear spaces, equivalent norms and its properties, finite dimensional normed linear spaces, quotient spaces, Banach spaces with examples like $\mathbb{R}^n, \mathbb{C}^n$, C[a, b] (with sup norm and integral norm), $c_0, l_p (1 \leq p \leq \infty)$, Riesz lemma and its applications in Banach spaces, series in Banach spaces, convergence of a series in Banach spaces. [7H]

Inner product spaces and Hilbert spaces with examples, continuity of inner product, Minkowski inequality, C-S inequality, basic results on inner product spaces and Hilbert spaces, parallelogram law, Pythagorean law, polarization identity, orthogonal and orthonormal vectors, complete orthonormal sets, separable Hilbert spaces, Bessel's inequality, Persevals'equality, Fourier expansion, $L_2[a,b]$ and its properties.

[12H]

Text Books:

1. Introductory Functional Analysis with Applications. E. Kreyszig (Wiley, 2007).
2. Elements of Functional Analysis. B.K. Lahiri (The world Press Pvt. Ltd., 1994).
3. Functional Analysis, A First Course. S. Kumaresan and D. Sukumar, 1st edition (Narosa, 2020).

Reference Books:

1. A first course in Functional Analysis. D. Somasundaram (Narosa, 2015).
2. A Course in Functional Analysis. J.B. Conway (Springer, 2007).
3. Functional Analysis. A.E. Taylor (John wiley and Sons, 1958).
4. Functional Analysis. B.V. Limaye, (New Age International Publications, 2017).
5. Functional Analysis. G. Bachman and L. Narici (Academic Press, 1966).

Group B

Elements of Multivariate Calculus (Marks - 25)

Objectives

To present the concepts of limit, continuity and derivative of functions from $\mathbb{R}^n$to $\mathbb{R}^m$.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. topological properties of $\mathbb{R}^n$, concepts of limit and continuity of functions from $\mathbb{R}^n$ to $\mathbb{R}^m$.
ii. derivative, directional derivative and partial derivative of functions from $\mathbb{R}^n$ to $\mathbb{R}^m$.
iii. matrix representation of derivative of functions from $\mathbb{R}^n$ to $\mathbb{R}^m$.
iv. implicit function theorem, inverse function theorem.

Skills: The students would be able to

i. determine limit, continuity, directional derivative, partial derivative and derivative of functions from $\mathbb{R}^n$ to $\mathbb{R}^m$.
ii. determine inverse of a vector valued function under certain conditions.
iii. find explicit form of some variable from an implicit form under certain conditions.

General competence: The students would gain

i. general idea of limit, continuity and derivative of functions from $\mathbb{R}^n$ to $\mathbb{R}^m$, which will be useful for further studies in multivariate calculus, functional analysis.
ii. analytical and reasoning skills which improve their thinking power.

Contents:

Topology of $\mathbb{R}^n$, functions from $\mathbb{R}^n$ to $\mathbb{R}^m$, projection functions, component functions, limit and continuity. [7H]

Derivative of functions from $\mathbb{R}^n$ to $\mathbb{R}^m$, directional derivatives and partial derivatives, partial derivatives of higher order, chain rule, matrix representation of derivative of functions, mean value theorem, continuously differentiable functions, C^∞-functions, real analytic functions. [13H]

Implicit function theorem, inverse function theorem (local and global). [5H]

Text Books:

1. Analysis on Manifolds. J. R. Munkres (Addison-Wesley Pub. Comp., 1991).
2.Calculus on Manifolds. M.Spivak (The Benjamin/Cummings Pub.comp.,1965).

Reference Books:

1.Introduction to Calculus and Analysis, Vol – II. R. Courant and F. John (Springer, 2004).
2. Basic Multivariate Calculus. T. Marsden (Springer, 2013).
3. Calculus, Vol. – II. T. M. Apostol (John Wiley Sons, 1969).

Course – MSMG203
Geometry of Curves and Surfaces (Marks - 50)

Total lectures Hours: 50H

Objectives

To study the local and global properties of curves in the plane and space, intrinsic properties of surfaces and their classifications and applications.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. curves in the plane and space.
ii. local and global properties of curves.
iii. geometric quantities like curvature and torsion.
iv. nature of various surfaces.
v. curvature of surfaces.
vi. isometry of surfaces.
vii. fundamental forms of surfaces.
viii. geodesics on surfaces.
ix. classification of surfaces
x. Gauss Theorema of Egregium with application

Skills: The students would be able to

i. explain the fundamental concepts of the differential geometry of curves and surfaces.
ii. demonstrate accurate and effective use of fundamental techniques of differential geometry.
iii. determine curvature, torsion for a space curve using Serret-Frenet formulas.
iv. achieve the concepts of invariant quantities of surfaces.
v. acquire geometrical problem-solving and analysis skills which may be applied to diverse situations in mathematics, physics, and engineering.

General competence: The students would gain

i. general idea of geometry of curves and surfaces which will be useful for further studies.
ii. Understanding Gauss-Bonnet theorem, which bridges the gap between topology and differential geometry.
iii. experience to learn geodesics of surfaces.

iv. concept of applications of Gauss Theorema of Egregium in cartography.

Contents:

Geometry of Curves: Definition of curves in R^n with examples, arc-length, reparametrization, level curves vs. parametrized curves, curvature of plane curves and space curves, properties of plane curves, torsion of space curves, properties of space curves, Serret-Frenet formulae, simple closed curves with periods, isoperimetric inequality, four vertex theorem (statement only). [12H]

Geometry of Surfaces: Definition of surfaces with various examples, smooth surfaces with examples, tangent, normal and orientability of surfaces, quadric surfaces, triply orthogonal systems, applications of the inverse function theory, first fundamental form, length of curves on surfaces, isometries of surfaces, conformal mapping of surfaces, surface area, equiareal maps and theory of Archimedes. [15H]

Curvature of surfaces, second fundamental form, curvature of curves on a surface, normal and principal curvatures, Euler's theorem, geometric interpretation of principal curvatures. The Gaussian curvature and mean curvature, pseudosphere, flat surfaces, surfaces of constant mean curvature, Gaussian curvature of compact surfaces, Gauss map. [11H]

Geodesics and its basic properties, geodesic equations, geodesic on surfaces of revolution, geodesic as shortest path, geodesic coordinates, minimal surfaces with examples, holomorphic functions. Gauss's theorem egregium. Codazzi Mainardi equation, third fundamental form, compact surfaces of constant Gaussian curvatures, Gauss-Bonnet theorem for simple closed curves and curvilinear polygons and for compact surfaces (statement only). [12H]

Text Book:

1. Elementary Differential Geometry. A. Pressley (Springer-Verlag,London, 2001 (Indian Reprint 2004)).

Reference Books:

1. Curves and Surfaces. S. Montel and A. Ros,2nd edition (American Mathematical Society, 2009).
2. Differential Geometry of Curves and Surfaces.M. P. Do Carmo (Prentice-hall, Inc., Englewood, Cliffs, New Jersey, 1976).
3. Elementary Differential Geometry. B. O' Neill, 2nd edition (Academic Press Inc., 2006).
4. Differential Geometry of curves and surfaces. V. A. Toponogov (Birkhauser, 2006).
5. Differential Geometry. E.Kreyszig (Dover Publications, Inc., New York, 1991).

Course – MSMG204
Differential Equations (Marks – 50)

Total lectures Hours: 50H

Objectives

To study

i. linear and non-linear differential equations through analytic and qualitative approaches.
ii. special functions and their properties.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. qualitative analysis of the differential equations.
ii. special functions and their use in different areas of mathematics.

Skills: The students would be able to

i. apply the solution techniques of the differential equations in different physical problems.
ii. solve the non-linear differential equation in qualitative approach.
iii. apply properties of special functions in different areas.

General competence: The students would gain

i. general idea of solution techniques of differential equations.
ii. understanding about the distinct features of linear and non-linear equations.
iii. experience to solve differential equations using special functions

Group A
Ordinary Differential Equations (Marks- 30)

Contents:

First-order equations: Existence and uniqueness theorems, Continuous dependence on initial conditions, Maximal interval of existence of solution, Maximal Gronwall's inequality. [3H]

Second order linear differential equations: Ordinary and singular points, regular and irregular singular points, Fuch's theorem (Statement only), Series solution, Frobenius Method, Special functions, Legendre and Hermite polynomials, Bessel function, Recurrence relations, Properties. [12H]

Boundary value problems: Construction of two-point boundary value problems, Sturm-Liouville equations, Green's functions, construction of Green's functions for both homogeneous and nonhomogeneous boundary conditions, eigenvalues and eigenfunctions. [5H]

System of ordinary differential equations: Homogeneous and non-homogeneous linear systems, Eigenvalue-Eigenvector methods, Fundamental matrix and its properties. [4H]

Nonlinear differential equations: Phase plane analysis: Autonomous systems, Quantative solutions, Phase portrait, Nature of fixed point and their stabilities, Linearization, Simple undamped pendulum and Lotka-Volterra Predator- prey models. [6H]

Group B
Partial Differential Equations (Marks - 20)

Contents:

Pre-requisite: Charpit's method, Jacobi's method, Integral surfaces, Method of characteristics, Monge cone, Characteristic strip. [3H]

Second order PDE in applied sciences: Origin of second order equations, Classification, Well-posedness, Variable separable methods, Dirichlet and Neumann problems.

Wave equation: Physical description of wave propagation and mathematical formulation, existence and uniqueness of solutions.

Heat conduction equation: Formulation of heat conduction equation, solutions of different types of initial-boundary conditions, existence and uniqueness of solutions.

Laplace and Poisson equations: Origin and physical descriptions of the equations, maximum and minimum principles, uniqueness of solutions. [10H]

Nonlinear partial differential equations: Concept of non-linearity, examples, Diffusion and dispersion processes, Formulations of Burger and KdV equations and their solutions. [7H]

Text Books:

1. An Introduction to Ordinary Differential Equations. E. A. Coddington (Dover, 1989).
2. Differential Equations. S. L. Ross, 3rd edition (Willey, 2007).
3. Elements of Partial Differential Equations. I. N. Sneddon (Dover, 2006)
4. An Elementary Course in Partial Differential Equations, T. Amarnath, 2nd edition (Jones and Bartlell, 2011).

Reference Books:

1. Differential Equations with Applications and Historical Notes. G. F. Simmons, 2nd edition (McGraw Hill Education, 2017).
2. Special Functions. E. D. Rainville (Macmillan, 1971).
3. Differential Equations and Dynamical Systems. Lawrence Perko, 3rd edition (Springer, 2004).
4. An Introduction to Dynamical Systems and Chaos. G. C. Layek (Springer, 2015).
5. Ordinary Differential Equations. G. Birkhoff & G. Rota, 4th edition (Wiley, 1989).
6. Special Functions of Mathematical Physics & Chemistry. I. N. Sneddon (Oliver & Boyd, 1980)

7. Green's Functions and Boundary Value Problems. I. Stackgold, 3rd edition (Wiley, 2011).
8. Partial Differential Equations, L. C. Evans, 2nd edition (American Mathematical Society, 2010).
9. Partial Differential Equations. P. Phoolan Prasad & R. Ravindran (Wiley, 1984).
10. Introduction to Partial Differential Equations. K. S. Rao, 3rd edition (Prentice Hall India, 2011).

Course Code: MSMG205
Operations Research (Marks – 50)

Total lectures Hours: 50H

Objectives

To introduce the basic concepts of Operations Research.

Learning outcomes

After completing the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. simplex method, revised simplex method, duality theory and dual simplex method for solving an L.P.P.
ii. transportation problem.
iii. the concept of post-optimality analysis.
iv. the concept of integer programming.
v. some preliminaries about inventory control problem.
vi. the concept of queueing theory.

Skills: The students would be able to

i. solve L.P.P. with the help of simplex method, revised simplex method, duality theory and dual simplex method.
ii. solve transportation problem.
iii. study the impact of post-optimality analysis.
iv. solve the integer linear programming problem.
v. formulate the deterministic inventory models.
vi. formulate queueing models.

General competence: The students would gain

i. the basic ideas about feasible and basic feasible solutions of L.P.P.

ii. the knowledge and understanding for different solution technique of several optimization problems.

Contents:

Prerequisite: Convex set and its properties [1H]
Simplex method& Revised simplex method [10H]
Duality theory and Dual Simplex method[5H]
Transportation problem: Mathematical formulation, Initial basic feasible solution, Optimal solution by u-v method. [4H]

Post Optimality Analysis: Changes in (i) objective function, (ii) requirement vector, (iii) coefficient matrix; Addition and deletion of variables, Addition and deletion of constraints. [9H]
Integer Programming: Gomory's cutting plane algorithm (All integer and mixed integer algorithms). [6H]
Inventory control: Basic EOQ and EPQ models (with and without shortage).[6H]
Queueing Theory: Basic features of queueing systems, operating characteristics of a queueing system, arrival and departure (birth & death) distributions, inter-arrival and service times distributions, transient, steady state conditions in queueing process. Poisson queueing models- M/M/1, M/M/C for finite and infinite queue length. [9H]

Text Books:

1. Operations Research – An Introduction. H. A. Taha(Prentice-Hall, 1997).
2. Advanced Optimization and Operations Research. A. K. Bhunia, L. Sahoo and A.A.Shaikh(Springer, 2020).
3. Operations Research: Theory and Applications. J. K. Sharma(Macmillan, 1997).

Reference Books:

1. Operations Research: Theory. Methods and Applications. S. D. Sharma & H. Sharma(Kedar Nath Ram Nath, 1972)
2. Operations Research. K. Swarup, P. K. Gupta & M. Mohan(Sultan Chand & Sons, 1978).
3. Introduction to Operations Research. F. S. Hillier, G. J. Lieberman(McGraw-Hill, 2001).
4. Nonlinear Programming-Theory and Algorithms. M. S. Bazaraa, H. D. Sherali, C. M. Shetty(Wiley-Interscience, 2006).
5. Introduction to Optimization Techniques: Fundamentals and Applications of Nonlinear Programming. M. Aokie (The Macmillan Company, 1971).

Course – MSMG206

Computer-aided Numerical practical (Marks: 50)

Total Practical Hours: 80H

Objectives

The main objectives of this course is to write C programming in order to solve different types of problems numerically with the help of computer.

Learning outcomes

After completing the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

v. syntax in C programming language
vi. writing in C programs
vii. different numerical methods,algorithms and their programmings

Skills: The students would be able to

vi. write C programs
vii. solve different problems numerically with the help of computer programming
viii. apply different numerical methods by using C programming to handle various types of Research problem in future

General competence: The students would gain

iii. the basic ideas about C programming
iv. the knowledge and understanding for different numerical techniques.

Sessional (Working formula, Algorithm & Program with output): 10 marks.
Viva Voce: 6 marks.
Pre-requisite:
C Programming: Operators, Decision making (branching and looping), Array, User defined functions, Reading and writing files.

Problems:

Group-A

Numerical Problem: 17 marks (Working formula and Algorithm: 5 marks, Program: 10 marks, result: 2 marks)

Numerical Solution of Polynomial Equations: Bairstow method.
Numerical Solution of a System of Equations: Matrix Inversion method, Successive-Over Relaxation.
Polynomial approximations: Hermite interpolation.
Methods for Finding Eigen Pair of a Matrix: Power method for finding least eigenvalue and corresponding eigenvector.
Integration: Romberg's method.

Group-B

Numerical Problem: 17 marks (Working formula and Algorithm: 5 marks, Program: 10 marks, result: 2 marks)
Numerical Solution of ODE:
Initial Value Problem: First order O. D. E. by Multistep Methods - Milne's method, Adams-Bashforth scheme- Adams-Moulton scheme, Second order O.D.E. by fourth order Runge-Kutta method.
Boundary Value Problem: Second order O.D.E. by finite difference method, Shooting method.
Numerical Solution of PDE by finite difference method: Explicit solution scheme for one-dimensional heat conduction equation, Crank-Nicholson scheme for one-dimensional heat conduction equation, Explicit solution scheme for one-dimensional wave equation.

Text Books:

1. C Language and Numerical Methods. C. Xavier (New Age International, 2007).
2. Numerical Recipes in C: The Art of Scientific Computing. W. H. Press, S. A. Teukolsky, W. T. Vetterling, B. P. Flannery (Cambridge University Press, 1992).

Reference Books:

1. Numerical Algorithms with C. G. Engeln-Müllges& F. Uhlig, 1stedition (Springer, 1996).
2. Numerical Methods in Science and Engineering.S.K. Pundir (CBS, 2017).
3. Numerical Computation using C. R. Glassey, 1stedition (Elsevier, 1992).

Course: MSMA301

Methods of Applied Mathematics (Marks – 50)

Total lectures Hours: 50H

Objectives

To present some important features of operator equations in Hilbert spaces and to apply those features for solving differential equations and integral equations.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. Generalised functions and generalised solutions.
ii. Important properties of the self-adjoint and compact operators.
iii. Existence and uniqueness of solution of operator equations in Hilbert spaces.
iv. Green's function and its properties.
v. Properties of solutions of parabolic, hyperbolic and elliptic equations.

Skills: The students would be able to

i. deal with integral equations more efficiently.
ii. Solve boundary value problems for PDE
iii. deal with the Green's function for various operators more effectively.
iv. Determine weak solution and generalised solutions of differential equations.

General competence: The students would gain

i. Knowledge on some important properties (in regard to application) of self-adjoint and compact operators.
ii. Fundamental ideas on the operator equations in Hilbert spaces.
iii. Advanced knowledge of Green's function and applications to solve BVP for PDE.
iv. knowledge about distribution theory.

Contents:

Theory of Distributions: Basic Ideas, Definitions, and Examples, Convergence of Sequences and Series of Distributions, Fourier Series, Fourier Transforms and Integrals, Differential Equations in Distributions, Weak Derivatives and Sobolev Spaces.[10H]

Operator equations on Hilbert Spaces: Self-adjointness, invertibility, boundedness and unboundedness of inverse operators, compactness of operators with illustrative examples, spectral value, eigenvalue problems, spectral theorem for compact self-adjoint operator (statement only), solvability of operator equations. [8H]

Integral equations with Hilbert-Schmidt kernel: Basic concepts, existence of solution, examples. [2H]

Boundary Value Problems (BVP): Existence, uniqueness of the solutions of BVP and continuous dependence on initial conditions; One-dimensional boundary value problems– BVP for equations of order p, BVP for second-order equations, well-posed and ill-posed problems. [3H]

Ordinary Differential Equations and Differential Operators: Adjoint of a differential operator; Regular Sturm-Liouville system – eigenvalues, equivalent integral equation; Inverse differential operators and Green's functions, Properties of Green's functions, Construction and uniqueness of Green's functions, Bilinear expansion of Green's function; solution of nonhomogeneous equation. [7H]

Multi-dimensional Green's functions: Multi-dimensional delta function; Green's functions for the Laplacian; Fundamental solution; Integral equation and Green's function. [3H]

Cauchy problem for linear partial differential equations: Basic concepts; Properties of linear PDE of order M in m variables; Cauchy problem; Solution criteria for second order PDE in two variables; Riemann method for solving Cauchy problem for linear hyperbolic PDE. [5H]

BVP for elliptic equations: Harmonic functions and its properties–mean value theorem, maximum-minimum principle, Boundary value problems–Dirichlet, Neumann, Robin, existence, uniqueness and stability of solutions of Dirichlet, Neumann, Robin problems, Dirichlet principle; Green's function for Dirichlet's problem of Laplace equation – properties, Method of images, Method of conformal mapping (two-dimensional). [5H]

BVP for parabolic equations: Heat equation in two independent variables; Solution of Cauchy problem using Dirac-delta function and Fourier transforms; Maximum-minimum principle; Stability condition. [4H]

Special techniques: Fourier transform technique – Green's function for the m-dimensional Laplacian, Helmholtz operator, wave equation; Eigen function expansion technique. [3H]

Text Books:

1. Green's Functions and Boundary Value Problems. I. Stackgold, 3rd edition (John Wiley & Sons, New York, 1979).
2. Generalized Functions, Volume 1: Properties and Operations. I. M. Gel'fand& G. E. Shilov (American Mathematical Society, 2016).
3. Introduction to Hilbert Spaces with Applications. L. Debnath& P. Mikusinski, 3rd edition (Academic Press, 2005).
4. Mathematical Physics. S. Hassani, 2nd edition (Springer, 2001).

Reference Books:

1. Methods of the Theory of Generalized Functions. V. S. Vladimirov (CRC Press, 2002).
2. Linear Systems and Operators in Hilbert Space. P. Fuhrmann (Dover Publications, 2014).
3. Methods of Theoretical Physics.P. M. Morse and H. Feshbach, Part I & II (McGraw-Hill Book Company, 1953).
4. Green's Function. G. F. Roach, 2nd edition (Cambridge University Press, 1982).

Course: MSMA302
Continuum Mechanics (Marks – 50)

Total lectures Hours: 50H

Objectives

To present a systematic introduction of the forces in continuum, deformation and motion, conservation laws and constitutive equations for a body in a continuum.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. spatial andmaterial coordinates.
ii. general stresses and strain in continuous materials.
iii. motion of continuum.

Skills: The students would be able to

i. apply the tensor formalism.
ii. describe motion of a body in continuum.
iii. apply the knowledge in solving the real world problems.

General competence: The students would gain

i. understanding about tensor calculus.
ii. ideas how to apply mathematical concepts in real world problem related to continuum mechanics.
iii. general idea of continuum mechanics which will be useful for further studies in specialized fields viz. elasticity, plasticity, continuum damage mechanics etc.
iv. ideas to appreciate a wide variety of advanced courses in solid and fluid mechanics.

Contents:

Prerequisite: Tensor, Transformation of Co-ordinates, Summation Convention, Kronecher delta, Invariant, Contravariant and Covariant tensors, Mixed tensors, Algebra of tensors, Symmetric and Skew-symmetric tensors, Contraction, Inner product of tensors, Quotient law. [6H]

Theory of strain: Body, Configuration, Deformation and flow, Material and spatial time derivatives, Lagrangian and Eulerian descriptions, Deformation gradient tensors, Finite strain tensor, Small deformation, Infinitesimal strain tensor,Principal strains, Strain invariants, Strain quadric, Compatibility equations for linear strains. [9H]

Theory of stress: Body and surface forces, Stress tensor, Stress components, Equations of equilibrium, Symmetry of stress tensor, Normal and shearing stresses, Maximum shearing stress, Principal stresses, Invariants of stress tensors, Stress quadric. [7H]

Motion of a continuum: Conservation of mass, Equation of continuity, Principle of conservation of momentum, Conservation of energy, First law of Thermodynamics. [4H]

Theory of elasticity: Ideal materials, Classical elasticity, Generalized Hooke's Law, Isotropic materials, Constitutive equation (stress-strain relations) for isotropic elastic solid, Strain-energy function, Beltrami-Michel compatibility equations for stresses, Field equations of linear elasticity, Fundamental boundary value problems of elasticity and uniqueness of their solutions (Statement only), Saint-Venant's principle. [9H]

Fluid media: Path lines, stream lines and streak lines, Bounding surface, Lagrange's criterion for bounding surface. [5H]

Water waves: General features, Condition at the free surface, Cisotti's equation, Complex potential, energy and path of particles for progressive waves and stationary waves, group velocity. [10H]

Text Books:

1. Mechanics of Continua. A. C. Eringen (Wiley, 1967).
2. Mathematical Theory of Continuum Mechanics. R. N. Chatterjee (Narosa Publishing House, New Delhi, 1999).
3. Tensor Analysis: Theory and Applications. I. S. Sokolnikoff (Wiley, 1951).

Reference Books:

1. Mathematical theory of Elasticity. I. S. Sokolnikoff, (Tata Mc Grow Hill Co., 1977).
2. Continuum Mechanics. D. S. Chandrasekharaiah and L. Debnath (Academic Press, 1994).
3. Tensor Calculus. M. C. Chaki (Calcutta Publishers, 2004).

Course Code: MSMA303

Theory of Electromagnetic fields and Special Theory of Relativity (Marks – 50)

Total lectures Hours: 50H

Objectives

i. To present the theories of Electric and magnetic field vectors.
ii. To introduce special theory of relativity

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. mathematical formalisms of electromagnetism
ii. Maxwell's laws
iii. Postulates of Special theory of relativity.
iv. Lorentz transformation.
v. Length construction and time dilation.
vi. Simultaneity
vii. Invariance of Maxwell's laws under STR.

Skills: The students would be able to

i. learn the techniques for solving Maxwell equation.
ii. solve dynamical problems involving high speeds.

General competence: The students would gain

i. general idea of electromagnetism
ii. understanding about fundamental relativistic constructions.

Contents:

Electrostatic field *E*: Coulomb law, Principle of superposition, Divergence and curl of ***E***, Boundary conditions; Electrostatic potential-Poisson equation, Energy in electrostatic field, Electric dipole, Conductor and insulator; ***E*** in dielectric media–electric polarization, Divergence of displacement vector, Energy in dielectric media. [5H]

Magnetostatic field*B*: Electric current, Equation of continuity, Ohm's law, Lorentz force law, Biot-Savart law, divergence and curl of ***B***, Boundary conditions; Magnetic vector potential – multi-pole expansion, Magnetic dipole; ***B*** in matter – magnetization, Auxiliary field ***H*** , Curl of ***H***. [6H]

Electromagnetic induction: Faraday's law; Inductance; Energy in magnetic field. [4H]

Maxwell's equations: Electrodynamics before Maxwell-Ampere-Maxwell equation; Maxwell's equations-in vacuum, in matter, Physical significance, Boundary conditions; Energy transfer and Poynting theorem. [5H]

Electromagnetic waves: Plane wave solution of Maxwell's equations-electromagnetic waves in vacuum; Reflection and transmission of plane electromagnetic waves at the boundary between two linear media. [5H]

General solution of Maxwell's equations: Electromagnetic potentials-gauging of potentials, Representation of fields in terms of potentials; Retarded potentials; Jefimenko's equations; Dipole radiation; Radiation by point charges. [5H]

Introduction to Special relativity: Maxwell's equation are not identical with Galelian transformations; Nature of light, Michelson Morley's experiment. [3H]

Postulates of special relativity**,** Lorentz transformations, Lorentz-Fitzgerald contraction, Lorentz invariants. [4H]

Consequences to time dilation, length contraction, relativistic simultaneity, relativistic Doppler effect, transformation of velocities; [3H]

Relativistic mass and energy; Force and acceleration in relativity; Lorentz group–boosts; [3H]

Four-vectors and tensors, light cone, time like light like and spacelike vectors; Relativistic particle dynamics. [3H]

Relativistic electrodynamics: Transformations of electric and magneic fields and invariance of Maxwell's equations; Relativistic Lagrangian. [4H]

Text Books:

1. Introduction to Electrodynamics. Griffiths D. J., 3rd edition (PHI, New Delhi, 2012).
2. Classical Mechanics with Introduction to Nonlinear Oscillation and Chaos. V. B. Bhatia (Narosa Publishing House, 1997).
3. The Special Theory of Relativity: A Mathematical Approach. Farook Rahaman , 1st edition (Springer, 2014)
4. Introduction to Special Relativity. Robert Resnick, 3rd edition (John Willey and Sons, 1968)

Reference Books:

1. Electricity. A. A. Coulson, 5thedition (Oliver and Boyd, Edinberg & London, 1974).
2. Relativity: The Special and the General Theory. A. Einstein (General Press, 2013).

Course Code: MSMA304 (GE)

Introduction to Operations Research (Marks – 25)

Total lectures Hours: 25H

Objectives

To introduce the preliminary concepts of Operations Research for the students of other disciplines.

Learning outcomes

After completing the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

(i) Simplex algorithm and duality theory.

(ii) the concept of unconstrained and constrained optimization problems with equality constraints, Lagrange's multiplier method and its physical interpretation.

(iii) the concept of game theory.

Skills: The students would be able to

(i) solve linear programming problem.

(ii) solve nonlinear unconstraint and equality constraint optimization problem.

(iii) solve different types game theory problem.

General competence: The students would gain

(i) general overview operations research and how to solve this type of problems.

Contents:

Simplex algorithm [7H]
Theory of duality [5H]
Unconstrained optimization[3H]
Constrained optimization with equality constraints- Lagrange's multiplier method, Interpretation of Lagrange multiplier.[5H]
Game theory: Formulation of two persons zero sum game, solution of two persons zero sum games with pure and mixed strategies. [5H]

Text Books:

1. Operations Research – An Introduction. H. A. Taha (Prentice-Hall, 1997).
2. Advanced Optimization and Operations Research. A. K. Bhunia, L. Sahoo & A.A.Shaikh (Springer, 2020).
3. Operations Research: Theory and Applications. J. K. Sharma (Macmillan, 1997).

Reference Books:

1. Operations Research: Theory, Methods and Applications. S. D. Sharma & H. Sharma (Kedar Nath Ram Nath, 1972).
2. Operations Research. K. Swarup, P. K. Gupta, M. Mohan (Sultan Chand & Sons, 1978).
3. Introduction to Operations Research, F. S. Hillier, G. J. Lieberman (McGraw-Hill, 2001).

Course: MSMA305-1

Boundary Layer Theory, Magneto-hydrodynamics-I (Marks: 50)

Total Lectures Hours: 50H

Objectives

To impart knowledge on

i. the principles of fluid flow and its significance

ii. flow behaviours and their applications for some particular cases

iii. boundary layer flows, governing equations of fluid flow for different flow regimes and different geometries

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. boundary layer theory, governing equations, limitation and significance.
ii. flow separation phenomena.
iii. self-similar solutions and their necessity.
iv. boundary layer parameters.

Skills: The students would be able to

i. find closed form solutions for some specific problems of viscous fluid flows governed by Navier-Stokes equations for different geometries.
ii. determine drag and lift forces acting on some fluid mechanical systems.
iii. analyze fluid flow problems governed by boundary layer equations for laminar flows.
iv. solve some physical flow problems using mathematical modeling.

General competence: The students would gain

i. general idea of fluid mechanics which will be useful for further study.
ii. understanding of fluid mechanical processes in nature.
iii. experience to construct mathematical modelsusing mathematical tools.

experience to find numerical solutions of specific fluid flow problems.

Contents:

Viscous flows: Continuum hypothesis, Fluid, Fluid viscosity, Newton's law of viscosity, Classification of fluids, Flow and its classification, Stokes' hypothesis, Stokes' law of friction, Navier-Stokes' equations and its dimensionless form, Reynolds number and its significance. [7H]

Some exact solutions of viscous fluid flows: Flow due to (i) suddenly accelerated plane wall, (ii) harmonic oscillation of plane wall, Flow around orthogonal stagnation–point and oblique stagnation-point, von-Karman flow near a rotating disk. [10H]

Slow motion: Creeping motion and its examples, Steady flow past a fixed sphere (Stokes' flow), Steady motion between parallel planes, Stokes' solution for slow steady parallel flow past a sphere, Stokes' paradox, Oseen's improvement over Stokes' solution, Oseen's solution for steady parallel flow past a sphere. [12H]

Boundary layer theory: Concept and properties of boundary layer flows, Prandtl's assumptions and validity, Boundary layer equations, Boundary layer parameters, Boundary layer separation. [6H]

Solutions of some boundary layer flows: Blasius boundary layer flows, Concept of self-similar flows, Blasius similar equation and series solution, Boundary layer flow in a convergent channel, Flow past a wedge, Planar jet flows. [10H]

Integral method for boundary layer equations: Basic concept of integral method, Karman's integral equation, Karman-Pohlhausen method and its applications for estimating boundary layer parameters. [5H]

Text Books:

1. Boundary Layer Theory. H. Schlichting (Springer, 2003).
2. Viscous Fluid Dynamics, J. L. Bansal, 2^{nd} edition (Oxford and IBH Publishing Co, 1977).
3. Physical Fluid Dynamics. D. J. Tritton, 2^{nd} edition (Oxford Science Publications, 1988).

Reference Books:

1. Foundations of Fluid Mechanics. S.W. Yuan (PHI, 1970).
2. Fluid Mechanics. P. K. Kundu (Academic Press, 1990).
3. An introduction to Fluid Dynamics. G. K. Batchelor (Cambridge University press, 1967).

Course: MSMA305-2

Turbulent Flows-I (Marks: 50)

Total Lectures Hours: 50H

Objectives

To developthe knowledge of laminar and turbulent flows and study the fundamental equations.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. continuum hypothesis applicable to viscous fluid flows.
ii. solution of Navier-Stokes equations for few special cases.
iii. empirical models for turbulent flows.

Skills: The students would be able to
i. gain the knowledge of viscous fluid flows.
ii. mathematical formulation of turbulent flows and modelling.

General competence: The students would gain
i. general idea of laminar and turbulent flows.
ii. solution techniques for boundary layer flows

Contents:
Viscous Fluid Dynamics: Continuum hypothesis, Fluid viscosity, Newton's law of viscosity, Stokes' law of friction for isotropic fluid medium, Constitutive equations for viscous fluid motion, Navier-Stokes equations of motion, boundary conditions, Reynolds number, High and low Reynolds number flows, Stokes' flow problems, Exact solutions of Navier-Stokes equations (steady flow between parallel plates, flow in a pipe, flow between concentric cylinders), Concept of boundary layer flows, Prandtl's assumptions and validity, Boundary layer equations, boundary layer parameters, Blasius boundary layer flow, Analysis of some laminar boundary layer flows (e.g., flow in convergent channel, jet flows), Flow separation phenomenon. [20H]

Turbulent Flows: Nature of turbulent motion, Statistical description of turbulent flow, Statistical symmetry, Self-preserving turbulence, Homogeneous and isotropic turbulence, Statistical stationary and stationary increments, Ergodicity and fully developed turbulence, Reynolds Averaged Navier-Stokes Equations, Reynolds stress tensor, Eddy viscosity, Closure problem, Phenomenological theories, Mixing length, Prandtl's momentum transfer theory, Taylor's vorticity transfer theory, Karman similarity hypothesis, velocity distribution in channel flow under constant pressure gradient. [20H]

Spread of Turbulence: Free-shear turbulent flows, Mixing layer between two parallel flows, Turbulent wake, Turbulent boundary layer flow in a planar jet, Turbulent flow through smooth circular pipe, Seventh power velocity distribution law. [10H]

Text Books:

1. Foundations of Fluid Mechanics. S. W. Yuan (PHI, 1970).
2. Turbulence: The Legacy of A. N. Kolmogorov, U. Frisch (Cambridge University Press, 1995).
3. Fluid Mechanics. P. K. Kundu (Academic Press, 1990).

Reference Books:

1. Boundary Layer Theory. H. Schlichting (Springer, 2003).
2. Physical Fluid Dynamics. D. J. Tritton, 2nd edition (Oxford Science Publications, 1988).
3. Turbulent Flows. S. B. Pope (Cambridge University Press, 2000).

Course: MSMA305-3
Space Sciences-I (Marks: 50)

Total Lectures Hours: 50H

Objectives

With the point of view of relativity, we deal with the fabric of space and time. This generalises the idea of Euclidean curvature to Ricci curvature and Newtonian idea of gravity is generalised to Einstein's field equations. Solution of this combined equation gives us the idea of black holes where, for the simplest one, they contain two singularities-one curvature singularity and one physical singularity. Some beautiful properties of these compact objects and observed. Astrophysically, these compact objects are formed of a catastrophic collapse of dense massive core of a dead star. If, however, the core has an intermediate amount of mass which fails to collapse by its own mass and a spinning neutron ball is left, the remnant is called a neutron star. Star with very small mass will leave a white dwarf instead. We will study all this remnants, stellar evolution, different epochs of their life etc in this course.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. mathematical formalisms of general relativity.
ii. Einstein field equation
iii. Black hole, neutron star and white dwarf.
iv. Fundamental accretion phenomena
v. Gravitational redshift.

Skills: The students would be able to

i. Justify perihelion precession, bending of light etc
ii. solve the Einstein field equation.
iii. Know the evolution of a star and its life.

General competence: The students would gain

i. general idea of relativistic view of universe.

ii. understanding about compact objects.
iii. Idea of Main sequence stars

Contents:

Introduction: The scope of the general theory of relativity, Geometry and physics; Space, time and gravity in Newtonian physics. [4H]

Tensor algebra and tensor calculus : Manifolds and coordinates–Curves and surfaces, Transformation of coordinates, Contravariant, covariant and mixed tensors, Elementary operations with tensors,The partial derivative of a tensor–Covariant differentiation and theaffine connection, The metric – Geodesics, Isometries–The Killing equation and conserved quantities,The Riemann tensor – The equation of geodesic deviation, The curvature and the Weyl tensors. [8H]

Principles of general relativity: The equivalence principle – The principle of general covariance – The principle of minimal gravitational coupling. [6H]

Field equations of general relativity: The vacuum Einstein equations; Derivation of vacuum Einstein equations from the action – The Bianchi identities; The stress-energy tensor – The cases of perfect fluid, scalar and electromagnetic fields; The structure of the Einstein equations. [8H]

Schwarzschild solution and black holes: The Schwarzschild solution – Properties of the metric – Symmetries and conserved quantities: Motion of particles in the Schwarzschild metric – Precession of the perihelion – Bending of light: Black holes – Event horizon, its properties and significance – Singularities: The Kruskal extension – Penrose diagrams [8H]

Friedmann-Lemaˆıtre-Robertson-Walker (FLRW) universe: Homogeneity and isotropy – The FLRW line-element; Friedmann equations – Solutions with different types of matter; Red-shift – Luminosity and angular diameter distances; The horizon problem – The inflationary scenario. [8H]

Gravitational waves: The linearized Einstein equations – Solutions to the wave equation – Production of weak gravitational waves; Gravitational radiation from binary stars – The quadrupole formula for the energy loss. [8H]

Textbooks:

1. The Classical Theory of Fields (Course of Theoretical Physics, Volume 2). L. D. Landau and E. M. Lifshitz, 4th edition (Pergamon Press, New York, 1975).
2. A First Course in General Relativity. B. F. Schutz, 2nd edition (Cambridge University Press, 1990).
3. Introducing Einstein's Relativity. R. d'Inverno, 2nd edition (Oxford University Press, 1992).

Reference Books:

1. Gravity: An Introduction to Einstein's General Relativity. J. B. Hartle (Pearson Education, Delhi, 2003).
2. Spacetime and Geometry. S. Carroll (Addison Wesley, New York, 2004).
3. General Relativity: An Introduction for Physicists. M. P. Hobson, G. P. Efstathiou and A. N. Lasenby, 1st edition (Cambridge University Press, 2006).
4. Gravitation and Cosmology. S. Weinberg, 3rd edition (John Wiley, New York, 1972).
5. Problem Book in Relativity and Gravitation. A. P. Lightman, W. H. Press, R. H. Price and S. A. Teukolsky, 2nd edition (Princeton University Press, 1975).
6. Relativity: Special, General and Cosmological. W. Rindler, 1st edition (Oxford University Press, 2006).
7. Gravitation: Foundation and Frontiers. T. Padmanabhan, 2nd edition (Cambridge University Press, Cambridge, 2010).
8. The Large Scale Structure of Spacetime. S. W. Hawking and G. F. R. Ellis, 3rd edition (Cambridge University Press, 1973).
9. Gravitation. C. W. Misner, K. S. Thorne and J. W. Wheeler, 2nd edition (W. H. Freeman and Company, 1973).
10. General Relativity, R. M. Wald, 2nd edition (The University of Chicago Press, 1984).
11. A Relativist's Toolkit. E. Poisson, 1st edition (Cambridge University Press, 2004).

Course: MSMA306-1
Advanced Optimization -I (Marks - 50)

Total lectures Hours: 50H

Objectives

To present a systematic introduction of the fundamentals of optimization and techniques.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. convex & concave functions and their properties.
ii. multivariate nonlinear optimization with equality and inequality constraints.
iii. one-dimensional optimization
iv. the theory of nonlinear programming.
v. numerical methods for unconstrained optimization.

Skills: The students would be able to

i. determine the convexity of an optimization problems

ii. solve the nonlinear optimization problems with equality and inequality constraints.
iii. solve one-dimensional optimization problem with interval uncertainty
iv. learn the theoretical developments of nonlinear optimization.
v. solve the unconstrained nonlinear optimization problems

General competence: The students would gain

i. fundamental idea about local and global optimality of different types of optimization problems.
ii. understanding about different techniques for solving constrained and unconstrained optimization problems.

Contents:

Convex and concave functions: Basic properties and some fundamental theorems of convex/concave functions, Differentiable convex and concave functions. [5H]
Unconstrained optimization: Single variable optimization, multivariate optimization. [4H]

Multivariable optimization with equality constraints: Method of substitution, Method of constrained variation, Lagrange's multiplier method, Interpretation of Lagrange multiplier, Multivariable optimization with inequality constraints: KKT conditions. [14H]

Quadratic Programming: Wolfe's modified simplex method, Beale's method. [5H]
Theory of nonlinear programming: Saddle point optimality criteria without differentiability, the minimization and the local minimization problems and some basic results, sufficient optimality theorem, Fritz John saddle point necessary optimality theorem, Slater's and Karlin's constraint qualifications and their equivalence, strict constraints qualification, Kuhn-Tucker saddle point necessary optimality theorem. [6H]
One-dimensional optimization: Function comparison method, Fibonacci and Golden section methods for unimodal functions. [6H]
Unconstrained optimization: Gradient methods, Steepest descent method, conjugate gradient method, Quasi-Newton's method, Daviddon-Fletcher-Powel method. [10H]

Text Books:

1. An Introduction to Optimization. Edwin K. P. Chang & S. Zak (John Wiley & Sons Inc., 2004).
2. Advanced Optimization and Operations Research. A.K. Bhunia, L.Sahoo & Ali Akbar Shaikh (Springer, 2019).

Reference Books:

1. Engineering Optimization Theory and Practice. Singiresu S Rao, 5th edition (John Wiley & Sons, Inc., 2020)
2. Introduction to Optimization Techniques: Fundamentals and Applications of Nonlinear Programming. M. Aokie (The Macmillan Company, 1971).

3. Mathematical Programming Techniques. N.S. Kambo (Affiliated East-West Press Pvt. Ltd., 2005).
4. Introduction to the Theory of Nonlinear Optimization. Johannes Jahn (Springer, 2007).
5. Non-Linear Programming. O. L. Mangasarian (McGraw Hill, 1994).
6. Optimization Techniques. C. Mohan & K. Deep (New Age Science, 2009).
7. Operations Research: Theory and Applications, J. K. Sharma (Macmillan, 1997).

Course: MSMA306-2
Advanced Operations Research-I (Marks - 50)

Total lectures Hours: 50H

Objectives

To impart the knowledge of Operations Research and different techniques for identifying appropriate solutions to particular problems**.**

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. the fundamental concepts of probability.
ii. the various deterministic and stochastic inventory control models
iii. mathematical models associated with project network.
iv. concept of information theory.
v. classical optimization techniques.

Skills: The students would be able to

i. apply and extend inventory models to analyse real world systems
ii. deal with the problems of project network.
iii. solve the optimization problems using classical optimization techniques.
iv. apply Shannon-Fano encoding procedure to obtain decodable code for a massage.

General competence: The students would gain

i. general idea of optimization technique which will be useful for solving various nonlinear and multilevel problems.
ii. experience to construct inventory theory models for real life problem
iii. idea of the network design for different project.

Contents:

Probability: Cumulative distribution function, joint cumulative distribution function, properties, probability mass/density functions, mathematical expectation, joint probability density functions, marginal and conditional distributions, moment generating function, characteristic function, expectation of function of two random variables, conditional expectations, independent random variables.

Distributions: Binomial, Poisson, Uniform, Normal, Exponential, Gamma, Weibull. [5H]

Inventory Control: Deterministic models: Multi-item EOQ model with constraints, EOQ models with price breaks, Probabilistic models: Probabilistic Inventory models, Newsvendor model and its extensions: Instantaneous and uniform demand with discrete and continuous cases; without and with lead time. [10H]

Project Network scheduling by PERT and CPM: Introduction, Basic difference between PERT and CPM, Steps of PERT/CPM Techniques, constructing project networks, PERT/CPM network components and precedence relationships, critical path analysis, Various types of floats and their significance, Project crashing, Probability in PERT analysis, Project Time-Cost. [12H]

Information Theory: Introduction, Communication Processes, memory less channel, the channel matrix, Probability relation in a channel, noiseless channel.

Measure of information, Entropy and its properties, Measure of other information quantities – – marginal and joint entropies, conditional entropies, expected mutual information, Axiom for an Entropy function, properties of Entropy function. Channel capacity, efficiency and redundancy, Encoding-Objectives of Encoding, Shannon-Fano Encoding Procedure. [12H]

Classical optimization techniques:

Unconstrained optimization: Single variable optimization, multivariate optimization.

Multivariable optimization with equality constraints: Lagrange's multiplier method, Interpretation of Lagrange multiplier, Multivariable optimization with inequality constraints: KKT conditions. [11H]

Text Books:

1. Operations Research: Theory, Methods and Applications. S. D. Sharma & H. Sharma (Kedar Nath Ram Nath, 2012).
2. Engineering Optimization: Theory and Practice, S. S. Rao, 5th edition (John Wiley & Sons, Inc., 2020).

Reference Books:

1. Operations Research – An Introduction. H. A. Taha, 10th edition (Pearson, 2017).
2. Operations Research: Theory and Applications. J. K. Sharma, 3rd edition (Macmillan, 2006).
3. Advanced Optimization and Operations Research. A. K. Bhunia, L. Sahoo & A. A. Shaikh (Springer, 2019).
4. Introduction to Operations Research, F. S. Hillier, G. J. Lieberman, 7th edition (McGraw-Hill, 2001).
5. A First Course in Probability. S. Ross, 9thedition (Pearson Education, 2013).

Course: MSMA306-3
Quantum Mechanics-I (Marks - 50)

Total lectures Hours: 50H

Objectives

To present a systematic introduction of the fundamentals of non-relativistic quantum mechanics, based on mathematical foundation.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. mathematical formalisms of quantum mechanics.
ii. Harmonic oscillator.
iii. conservation laws, symmetries, angular momentum and spin.
iv. structural properties of one-electron atoms.
v. approximate solutions of the time-independent Schrodinger equation.

Skills: The students would be able to

i. apply the principles of quantum mechanics to calculate some important observables for a given wave function.
ii. solve the time-dependent Schrodinger equation for some model potentials.
iii. combine spin and angular momenta.
iv. determine bound state energies and wave functions approximately for some model potentials and two-electron atoms/ions.

General competence: The students would gain

i. general idea of non-relativistic quantum mechanics which will be useful for further studies in theoretical physics.
ii. understanding about fundamental quantum mechanical processes in nature.
iii. experience to construct approximate quantum mechanical modelsusing mathematical tools

Contents:

Fundamental ideas of quantum mechanics: Nature of the electromagnetic radiation; Wave-article duality-double-slit experiment, quantum unification of the two aspects of light, matter waves; Wave functions and Schrodinger equation; Quantum description of particle-wave packet, uncertainty relation. [6H]

Mathematical formalism of quantum mechanics: Wave function space-bases, representation; State space-bases, representation; Observables-R and P observables; Postulates of quantum mechanics. [5H]

Physical interpretation of the postulates: Statistical interpretation-expectation values, Ehrenfest theorem, uncertainty principle; Physical implications of the Schrodinger equation-evolution of physical systems, superposition principle, conservation of probability, equation of continuity; Solution of the Schrodinger equation-time evolution operator, stationary state, time-independent Schrodinger equation; Equations of motion-Schrodinger picture, Heisenberg picture, interaction picture. [7H]

Theory of harmonic oscillator: Matrix formulation-creation and annihilation operators; Energy values; Matrix representation in |n> basis; Representation in the coordinate basis; Planck's law; Oscillator in higher dimensions. [5H]

Symmetry and conservation laws: Symmetry transformations-basic concepts, examples; Translation in space; Translation of time; Rotation in space; Space inversion; Time reversal. [5H]

Angular momentum: Orbital angular momentum-eigen values and eigen functions of L^2 and L_z; Angular momentum operators $\boldsymbol{J}$– commutation relations, eigen values and eigen functions; Representations of the angular momentum operators. [5H]

Spin: Idea of spin-Bosons, Fermions; Spin one-half-eigen functions, Pauli matrices; Total Hilbert space for spin-half particles; Addition of angular momenta; Clebsch-Gordan coefficients-computaion, recursion relations, construction procedure; Identical particles-symmetrisation postulate, Pauli exclusion principle, normalization of states. [5H]

One-electron atom: Schrodinger equation; Energy levels, Eigen functions and bound states, Expectation values and virial theorem; Solution in parabolic coordinates; Special hydrogenic atom (brief description)-positronium, muonium, antihydrogen, Rydberg atoms. [4H]

Time-independent perturbation theory: Basic concepts; Derivation-up to the second order correction to the energy values and wave functions; Applications-anharmonic oscillator; normal helium atom, ground state of hydrogen and Stark effect. [4H]

Variational method: Rayleigh-Ritz variational principle; Applications-one dimensional harmonic oscillator, hydrogen atom, helium atom. [4H]

Text Books:

1. Quantum Mechanics Vol. 1. C. Cohen-Tannoudji, B. Diu & F. Laloe (Wiley-Interscience publication, 1977).
2. Lectures on Quantum Mechanics. A. Das (Hindusthan Book Agency,New Delhi, 2003).

Reference Books:

1. Quantum Mechanics. B. H. Bransden & C. J. Joachain (Prentics Hall, 2005).
2. Physics of Atoms and Molecules. B. H. Bransden, A. Bransden & C. J. Joachain (Pearson Education, 2007).
3. Introduction to Quantum Mechanics. D. J. Griffiths (Pearson Prentics Hall, 2005).
4. Quantum Mechanics. L. I. Schiff (McGraw-Hill, New York, 1968).

Course: MSMA306-4
Fuzzy Mathematics and Applications-I (Marks - 50)

Total lectures Hours: 50H

Objectives

To present a systematic introduction of operations of fuzzy mathematics and applications.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. fuzzy sets, fuzzy numbers, fuzzy relations and fuzzy matrix.

Skills: The students would be able to

i. apply fuzzy numbers, fuzzy relations and fuzzy matrix in real life decision making problem with imprecise parameters.

General competence: The students would gain

i. fundamental idea about application of fuzzy mathematics in different types of problem.

Contents:

Definition of Fuzzy sets. Alpha-cut. Normality. Extension Principle. Basic Operations like inclusion. Completion, Union and intersection, Difference. [12H]

Operations on Fuzzy sets: Extension principle for fuzzy sets, fuzzy compliments, t-norms and t-conorms, Definition of intersection and union by Hamacher, Yager's union and intersection of two fuzzy sets, intersection and union of two fuzzy sets as defined by Dubois and Prade, Combination of operations, Aggregation operations. [12H]

Fuzzy numbers: Interval, Triangular and trapezoid fuzzy numbers. Addition, Subtraction, Multiplication, Division, other arithmetic. [7H]

Fuzzy relations: Introduction, Projections and cylindrical fuzzy relations, Composition, properties of Min-max composition, binary relations and their compositions, compatibility relation, Fuzzy equivalence relations, fuzzy ordering relation, Fuzzy morphisms. [12H]

Fuzzy matrix theory: operations, convergence, power convergence, nilpotent fuzzy matrix. Rank of fuzzy matrix. [7H]

Text books:

1) Fuzzy sets and fuzzy logic. G.J. Klir & B Yuan (Prentice Hall of India Ltd. New Delhi 1997).

Reference books

1) Fuzzy Set Theory and its Applications. H. J. Zimmermann (Allied Publishers Ltd. New Delhi 1991).

Course: MSMA307
Community Engagement Activities (Marks - 25)

Each student is to carry out some work related to the development/welfare of a community/society as per the guidelines of the Department to be framed from time to time. Marks of the Internal Assessment will awarded through viva-voce and marks of the term-end examination will be awarded through the evaluation of the report to be submitted by the respective student.

MSMP301
Abstract Algebra (Marks - 50)

Total lectures Hours:50H

Objectives

To present a systematic introduction of the fundamentals course on abstract algebra.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. Importance of solvable series and its motivation related to investigate solution by radical of a polynomial over $\mathbb{R}$ of degree ≥ 5.

ii. Application of extension of field to generalized version of fundamental theorem of algebra.
iii. A brief coverage on Galois theory, one of the most fundamental concepts in abstract algebra
iv. The reason behind the impossibility of doubling a cube using straight edge and compass by the technique of algebraic extension.
v. The basic contents of module theory which is indeed a gateway of the most burning subject, namely algebraic geometry.

Skills: The students would be able to

i. Compute many problems like insolvability of $S_n\ for\ n \geq 5$.
ii. Construct algebraic closure of every field and many examples and counter examples.
iii. Explain whether a spitting field of a polynomial is Galois and to study the structural properties of group of automorphisms of any field of characteristic 0.
iv. To manipulate free basis of a modules and to study importance of exact sequences in algebra and to enlighten in details the structures of groups and rings.

General competence: The students would gain

i. general idea of constructing algebraic, separable and normal extension of a field
ii. to compute Galois group of a polynomial over a field.
iii. Expertized in solving many tricky problems in algebra

Contents:

Groups and Rings: Normal series, subnormal series, solvable series, solvable groups, composition series, nilpotent group, Jordan-H$\ddot{o}$lder theorem, ascending central series, descending central series, commutator subgroup, insolvability of $S_n, n \geq 5$,Noetherian ring and Artinian ring-examples and properties, Hilbert basis theorem. [10H]

Field Extensions: Algebraic extensions, transcendental extensions, degree of extensions, simple extensions, finite extensions, simple algebraic extensions, minimal polynomial of an algebraic element, isomorphism extension theorem, splitting fields, fundamental theorem of general algebra (Kronecker theorem), existence theorem, isomorphism theorem, algebraically closed field, existence of algebraically closed field, algebraic closures, existence and uniqueness (up to isomorphism) of algebraic closures of a field, field of algebraic members, separable extension, separable and inseparable polynomials, separable and inseparable extensions, perfect field, Artin's theorem, finite field, structure of finite field, existence of GF (p^n), construction of finite fields, field of order p^n, $p$rimitive elements, normal extensions, automorphisms of field extensions, Galois extensions, fundamental theorem of Galois theory, solutions of polynomial equations by radicals, insolvability of general polynomial equation of order 5 by radicals, roots of unity, primitive roots of unity, Cyclotomic fields, Cyclotomic polynomial, Wedderburn's theorem, geometric constructions by straightedge and compass only. [25H]

Modules: Modules and module homomorphisms, submodules and quotient modules, operations on submodule, direct sum and product, faithful module, finitely generated modules, Nakayama's lemma, free modules, basis of free module, exact sequences, split-exact sequence, five lemma, tensor product of modules, restriction and extension of scalars, exactness properties of the tensor product, algebras, tensor product of algebras. [15H]

Text Books:

1. Introduction to commutative algebra. M. Atiya & I.G. MacDonald, 1st edition (Indian) (Levant Books, India, 2007).
2. Abstract Algebra. D.S. Dummit & R.M. Foote, 3rd edition (Wiley, 2016).
3. Basic Abstract Algebra. P. B. Bhattacharya, S. K. Jain& S. R. Nagpaul, 2nd edition (Cambridge University Press, 2014).

Reference Books:

1. The Theory of Rings. N. H. McCoy, 1st edition (Chelsea Publishing Company, 1973).
2. Module Theory: An Approach to Linear Algebra. T. S. Blyth, 1st edition (Clarendon Press, 1977).
3. Algebra.T. W. Hungerford, 3rd edition (Cengage Learning India Pvt Ltd,2012)
4. A First Course in Abstract Algebra. J.B. Fraleigh, 7th edition (Pearson Education (India) 2002).
5. Fundamental of Abstract Algebra. D.S. Malik, J.N. Mordenson &M.K.Sen, 1st edition (Mcgraw-Hill Book Company – Koga, 1997).

Course: MSMP302

Analysis – I (Marks: 50)

Total Lectures Hours: 50H

Group A

Topological Vector Spaces (Marks - 25)

Objectives

To familiarize the students with the systematic study of topological properties on an arbitrary vector space that might help the students to get an exposure on the basic concepts of a core area namely topological vector space under the banner of functional analysis.

Learning outcomes

Knowledge:

The students learn from the study of topological vector spaces on the basic ideas namely

(i) fundamental system of neighborhoods of zero vector in the underlying space

(ii) topological and algebraical properties:
   (a) homeomorphism
   (b) separation property
   (c) connectedness property
   (d) compactness and local compactness property
   (e) dimensionality of underlined space

Skills:

- The students are motivated by experiencing their knowledge on such area that facilitates them to work out various problems on the allied areas of functional analysis.
- The students are influenced to identify the analogue properties and distinct properties of Euclidean spaces even normed spaces also.

General Competence:

(i) Students who has had a course of advanced calculus and a minimum of algebra and topology finds no difficulty on this course
(ii) It helps the students to read and to learn evaluate critically further topics in locally convex topological vector space

It motivates the students to make them easier in exercising the use of functional analysis in optimization theory, control theory and mathematical economics etc.

Contents:
Convex sets, convex hull, representation theorem for convex hull. [3H]
Symmetric sets, balanced sets, absorbing sets, bounded sets, absolutely convex sets and their properties. [4H]
Topological vector spaces, closed sets, open sets with its properties, neighbourhoods, local bases and its properties, separation properties of a topological vector space. [9H]
Linear operators with its continuity properties on topological vector spaces and homeomorphism in topological vector spaces.
[3H]
Connectedness, compactness and locally compactness of a topological vector space and its properties on finite dimensional topological vector spaces. [6H]

Group B
Operator Theory (Marks - 25)

Objectives:

To present a systematic introductory knowledge on operator theory as a part of functional analysis.

Learning outcomes: On completion of the course the students should have the following outcomes defined in terms of knowledge, skill and general competence.

Knowledge: The student will acquire knowledge on

i) different kinds of operators like closed linear operators, conjugate operators, adjoint operators, closure of a linear operator, isometric operators, normal operators, Hermitian operators, symmetric operators, self adjoint operators, sesquilinear functionals etc.
ii) relation between the operators
iii) fundamental properties of such operators
iv) generalization of CS inequality in terms of sesquilinear functionals.

Skills: The students will be able to

i) apply the fundamental results to solve various problems on operators.
ii) correlate the classical results on matrices with linear operators defined on finite dimensional spaces
iii) compare a particular result for different kinds of operators.

General competence: The students will gain

i) general ideas on operators with the underlying domain of definition
ii) overall knowledge on the properties that an operator possesses.

Contents:

Closed linear operators and its properties, bounded inverse theorem (statement only), closed graph theorem. [4H]

Adjoint (conjugate) operators over normed linear spaces and their algebraic properties, annihilators and properties of annihilators of bounded linear operator on a normed linear space, closure of a linear operator. [6H]

Adjoint (Hilbert adjoint) operators on inner product spaces and Hilbert spaces, relationship between closure and adjoint operators, relationship between conjugate and adjoint operators. [4H]

Normal operators, isometric operators, unitary operators and their properties. [3H]

Hermitian, symmetric and self-adjoint operators and their properties on inner product spaces and on Hilbert spaces. [4H]

Sesquilinear functionals on linear spaces and on Hilbert spaces, generalization of Cauchy-Schwarz inequality. [4H]

Text Books:

1. Elements of Functional Analysis. B.K. Lahiri (The world Press Pvt. Ltd., Kolkata, 1994).
2. Functional Analysis. A.E. Taylor (John Wiley and Sons, New York,1958).
3. Functional Analysis. W. Rudin (TMG Publishing Co. Ltd., New Delhi,1973).

4. Functional Analysis. G. Bachman & L. Narici (*Academic* Press, 1966).
5. Introductory Functional Analysis with Applications. E. Kreyszig (Wiley Eastern, 1989).

Reference Books:

1. Topological Vector spaces. L. Narici& E. Beckenstein (Marcel Dekker Inc, New York and Basel,1985).
2. Topological Vector spaces and Distributions, Vol. – I, J. Horv*á*th (Addision-Wesley, 1966).
3. Functional Analysis. B. V. Limaye (Wiley Eastern Ltd, New Delhi, 1981)
4. Functional Analysis. M. T. Nair (Prentice-Hall of India Pvt. Ltd, New Delhi, 2002)
5. Functional Analysis. K. Yosida, 3rd edition (Springer Verlag, New York, 1990).

Course: MSMP303
Geometry of Manifolds (Marks-50)

Total Lectures Hours: 50H

Objectives

To introduce the concept of smooth manifolds with various examples along with their geometrical properties, exterior algebra and exterior derivative, Lie group and Lie algebra.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain the knowledge about

i. topological manifolds and smooth manifolds.
ii. tangent space and cotangent space.
iii. vector fields on manifolds.
iv. integral curves on manifolds.
v. pull back and push forward maps on manifolds.
vi. Lie bracket and its properties.
vii. submanifolds.
viii. tensors and tensor fields on manifolds.
ix. exterior algebra and exterior derivative on manifolds.
x. Lie groups and Lie algebras.

Skills: The students would be able to

i. establish concrete properties of smooth manifolds through calculations and theory.
ii. construct smooth manifolds, Lie groups.
iii. extend the notion of bump functions to arbitrary closed subsets of manifold.

iv. use partitions of unity to construct a special kind of smooth functions, known as exhaustion function.
v. establish the applications of manifolds in mechanics.

General competence: The students would gain

i. general ideaof smooth manifolds with various examples whichwill be useful for further studies in general theory of relativity and cosmology.
ii. the concept of abstraction, generalization, existence, characterization, classification and applications of the manifolds in various branch of sciences.
iii. the most important properties of manifolds as global analysis in mathematics and applications in mathematical physics, dynamical systems and modelling.
iv. the concept of Lie groups which are widely used in various branches of modern mathematics and physics.
v. the theory of exterior algebra and exterior differentiation systematically, which is an one of the indispensable tools in the study of manifold theory.

Contents:

Manifolds:Topological manifolds with examples, coordinates charts, topological properties of manifolds, smooth structures, smooth manifolds with examples, examples of non-Hausdorff, non-connected, non-second countable manifolds, manifolds with boundary. [10H]

Smooth Functions and Smooth Maps: Smooth maps between manifolds, diffeomorphisms on manifolds and its properties, partitions of unity and its applications.

Tangent Vectors: Various definitions of tangent vectors, the differential of a smooth map, computation in coordinates, tangent spaces, tangent bundle, covectors, cotangent spaces, cotangent bundle**,** pushforward and pullback maps. [10H]

Submanifolds: Maps of constant rank, submersions, immersion and embeddings, embedded submanifolds, immersed submanifolds (definitions and examples only), rank theorem (statement only).

Vector Fields on Manifolds: Vector fields on manifolds, local and global frames, smoothly related vector fields, Lie brackets and its properties, integral curves and flows, local and global 1-parameter group of transformations, complete vector fields, distributions, integral manifolds, application of Frobenius theorem. [10H]

Exterior Algebra and Exterior Derivatives: Multilinear algebra, tensors**,** tensor products, symmetric and alternating tensors, tensors and tensor fields on manifolds (definition and examples), wedge product and exterior algebra, differential forms on manifolds, exterior derivatives. [10H]

Lie Groups and Lie Algebra: Definition and examples of Lie groups, Lie algebra of Lie groups, Heisenberg groups, Maurer-Cartan structure equation, structure constants, Lie group homomorphisms and isomorphisms, Lie subgroups (definition and examples, characterization without proof), 1-parameter subgroups and exponential maps, Lie derivatives (definition and examples). [10H]

Text Books:

1. Introduction to Smooth Manifolds. John M. Lee, 2nd edition (Springer-Verlag, 2012).

Reference Books:

1. An introduction to manifolds. L. W. Tu (Springer, 2007).
2. Differential Geometry of Manifolds.U. C. De &A. A. Shaikh (Narosa Publ. Pvt. Ltd, New Delhi, 2007).
3. A course in Differential Geometry and Lie groups. S. Kumaresan (Hindustan Book Agency, 2002).
4. An Introduction to Differentiable Manifolds and Riemannian Geometry. William H. Boothby (Academic Press, New York, 1975).
5. Foundations of Differential Geometry, Vol. 1. S. Kobayashi & K.Nomizu (Inter science Press, New York, 1969).
6. Introduction to Differential Manifolds,S. Lang (John Wiley and Sons, New York, 1962).
7. Differential Geometry, Lie Groups and Symmetric spaces,S. Helgason (AcademicPress, New York, 1978).
8. Structures on Manifolds, K. Yano & M. Kon (World Scientific Publishing Company, Singapore, 1984).

Course: MSMP304 (GE)
Introduction to Graph Theory (Marks - 25)

Total Lecture Hours: 25H

Objectives

To understand and apply the fundamental concepts of graph theory based on tools in solving practical problems.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. undirected and directed graphs.
ii. Eulerian graphs.
iii. various characterizations of trees with applications.
iv. bipartite graph and its characterization.
v. planar graphs.
vi. matrix representation of graphs.

Skills: The students would be able to

i. assimilate various graph theoretic concepts and familiarize with their applications.
ii. be efficient in handling with discrete structures.
iii. be efficient in notions of matrix representation of graph, planarity.
iv. be efficient in solving concrete graph colouring problems.
v. solve real world problems that can be modelled by graphs.

General competence: The students would gain

i. general idea of graph theory and its real life applications.
ii. understanding about graphic sequence.
iii. experience to apply Euler's formula.
iv. ability to use graphs for various map colouring problems.
v. idea about the application of graphs in computer science.

Contents:

Graphs: Undirected graphs, directed graphs, basic properties of graphs, walks, paths, cycles, connected graphs, components of a graph, complete graph, complement of a graph, bipartite graphs, characterization a bipartite graph. [10H]
Euler graph: Königsberg bridge problem, Euler graph and its characterization. [5H]
Planar graphs: Planar graphs and non-planar graphs, face-size equation, Euler's formula for a planar graph. The graphs K_5 and $K_{3,3}$, Kuratowski's theorem (statement only). [5H]
Trees: Trees with basic properties and characterizations, spanning tree, rooted tree, binary tree, minimal spanning tree, Kruskal's algorithm. [5H]

Text Book:

1. Graph Theory. Nar Sing Deo (Prentice-Hall, 1974).

Reference Books:

1. A First Look at Graph Theory. J. Clark & D. A. Holton (Allied Publishers Ltd., 1995).
2. Introduction to Graph Theory. D. S. Malik, M. K. Sen & S. Ghosh (Cengage Learning Asia, 2014).
3. Introduction to Graph Theory. Douglas Brent West (Prentice Hall, 2001).
4. Graph Theory. Frank Harary (Addison-Wesley, 1971).

Course: MSMP305-1
Advanced Functional Analysis -I (Marks - 50)

Total Lectures Hours: 50H

Objectives

As the students with already have exposures on the course topological vector spaces, the course is designed in such a way that student can learn also on the fundamental topics of one of the core areas namely locally convex topological vector spaces of functional analysis and the students can pursue their higher studies in more advanced areas of functional analysis. The power of abstraction in mathematics can be realized from the concept of elementary results of functional analysis.

Learning outcomes

Knowledge:

The students learn

- the basic ideas namely: convexity structure of Banach spaces and Hilbert spaces
- the basic ideas of
  - (i) locally convex topological vector spaces together with the study of their completeness property
  - (ii) criterion for normability of locally convex topological vector spaces
  - (iii) Minkowski functionals and semi-norms together with their characteristic properties
  - (iv) the four fundamental results of functional analysis namely Hahn Banach theorem, uniform boundedness principle (Banach Steinhaus theorem), open mapping theorem, closed graph theorem on locally convex topological vector spaces together with their applications
  - (v) hyperplane and separation properties of locally convex topological vector spaces
  - (vi) the theory of Barreled spaces and Bornological spaces and its consequences

Skills:

- Students who has had a course of advanced calculus and a minimum of algebra and topology finds no difficulty on this course
- The students are motivated by experiencing their knowledge on such area that facilitates them to work out various problems on the allied areas of functional analysis.
- The students are influenced to identify the analogue properties and distinct properties of Euclidean spaces even also in normed spaces etc.

General Competence:

(i) It helps the students to read and to learn evaluate critically further topics in locally convex topological vector space

(ii) It motivates the students to make them easier in exercising the applications of such coursein distribution theory, optimization theory, control theory and mathematical economics etc.

Contents:

Strict convexity and uniform convexity of a Banach space and Hilbert space with examples, properties of strictly convex and uniformly convex Banach spaces. [6H]

Clarkson inequality, Clarkson's renorming lemma, Milman and Pettit's theorem (only statements), reflexivity of a uniformly convex Banach space. [6H]

Locally convex topological vector spaces: Filters, convergence of filters, ultrafilter, fundamental system of neighbourhoods, criterion for a locally convex topological vector space, completeness, property, Frechet space, quotient spaces, Minkowski functionals, semi-norms and its characteristic properties, criterion for normability, Kolmogorov theorem, metrizability, Hahn Banach theorem, uniform boundedness principle (Banach Steinhaus theorem), open mapping theorem, closed graph theorem. [24H]

Hyperplane and its separation property. [6H]

Barrelled spaces and Bornological spaces with examples, criterion for locally convex topological vector spaces to be Barreled and Bornological. [8H]

Text Books:

1. Topological Vector spaces and Distributions, Vol.-I. J. Horvoth (Addison-Wesley Publishing Co., 1966).
2. Functional Analysis. B.V. Limaye (New Age International Publications, 2017).

Reference Books:

1. Topological Vector Spaces. A.A. Schaffer (Springer , 2^{nd}Edn., 1991).
2. Geometry of Banach Spaces. J. Diestel (Springer, 1975).

Course – MSMP305-2

Advanced Differential Geometry-I (Marks-50)

Total Lectures Hours: 50H

Objectives

To present a systematic study of the basic local and global properties of Riemannian manifolds and their submanifolds.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. Riemannian manifolds
ii. fundamental theorem of Riemannian geometry and Riemannian connection.
iii. several curvature tensors on Riemannian manifolds
iv. Einstein manifolds and its generalizations.
v. geodesics on Riemannian manifolds.
vi. various types of submanifolds of Riemannian manifolds.
vii. some transformations on Riemannian manifolds and their invariants.

Skills: The students would be able to

i. apply the Koszul formula to calculate the covariant derivative of a vector field with respect to a vector field.
ii. compute the curvature tensor, Ricci curvature, scalar curvature.
iii. learn the properties of some interesting curvature tensors.
iv. determine semi-symmetric metric connections and quarter symmetric metric connections on Riemannian manifolds.

General competence: The students would gain

i. general idea of Einstein manifolds and their generalizations which will be useful for further studies in general relativity and cosmology.
ii. understanding about geodesics on Riemannian manifolds.
iii. about basic curvature identities of Riemannian manifolds.
iv. experience to construct different types of submanifolds.

Contents:

Riemannian manifolds: Affine connections, torsion tensor and curvature tensor of an affine connection, Riemannian metrics, Riemannian manifold, fundamental theorem of Riemannian geometry, Riemannian connection, Bianchi identities, generalized and proper generalized curvature tensors, Ricci tensor, scalar curvature, Gaussian curvature, sectional curvature, Schur's theorem, isometry groups of Riemannian manifold, model spaces of Riemannian geometry, Einstein manifolds, quasi-Einstein manifolds and their generalizations. [20H]

Transformations on Riemann manifolds: Geodesics on Riemannian manifolds, Hopf-Rinow theorem, conformal transformations, projective transformations, concircular transformations, conharmonic transformations and their properties, semi-symmetric and quarter symmetric metric connections. [15H]

Theory of submanifolds: Submanifolds and hypersurfaces of Riemannian manifolds, induced connection and second fundamental form, Gauss and Weingarten formulae, equations of Gauss, Codazzi and Ricci, mean curvature, totally geodesic and totally umbilical submanifolds, minimal submanifolds. [15H]

Text Books:

1. Riemannian Geometry. P. Petersen (Springer-Verlag, 2016).
2. Geometry of Submanifolds. B. Y. Chen (Dover Publications Inc., 2019).

Reference Books:

1. Riemannian Manifolds, An Introduction to Curvature. J. M. Lee (Springer-Verlag, 2005).
2. Differential Geometry of Manifolds. U. C. De & A. A. Shaikh (Narosa Publ. Pvt. Ltd, New Delhi, 2007).
3. Foundations of Differential Geometry, Vol. 2. S. Kobayashi & K. Nomizu (Interscience Press, New York, 1969).
4. Riemannian Geometry. T. J. Willmore (Oxford University Press, 1997).
5. Structure on Manifolds, K. Yano & M. Kon (World Scientific Publication, Singapore, 1984).
6. Riemannian Geometry. Manfredo P. Do Carmo (Birkhauser, Boston, 1992).

Course: MSMP305-3
Advanced Complex Analysis-I (Marks - 50)

Total Lectures Hours: 50H

Objectives

To present a systematic introduction on the detailed study on complex analysis with the presentation of the detailed theory of entire functions, Dirichlet series and their important properties.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

i. On the deeper aspects of complex analysis
ii. Ontesting the convergence and absolute convergence of the Dirichlet series
iii. On the study of Riemann zeta function.
iv. On the study of entire functions with its growth and distribution of their zeros.
v. On the study of the growth of an entire function and distribution of its zeros.
vi. On Weierstrass factor theorem
vii. On concepts of canonical product, Hadamard's factorization theorem and Picard's theorems.
viii. On further applications in various branches of mathematical sciences.

Contents:

The functions M(r) and A(r), theorems of Borel and Caratheodary, convex functions and Hadamard three-circle theorem, open mapping theorem. [10H]

Dirichlet series, abscissa of convergence and abscissa of absolute convergence, their representations in terms of the coefficients of the Dirichlet series, Riemann zeta function, the product development of and the zeros of the zeta functions. [15H]
Entire functions, growth of an entire function, order and type and their representations in terms of the Taylor's coefficients, distribution of zeros, Picard's first theorem, Weierstrass factor theorem, the exponent of convergence of zeros, canonical product, Hadamard's factorization theorem, Borel's theorem, Picard's second theorem. [25H]

Text Books:

1. Theory of Functions of a Complex Variables,Vol. I & II. A. I. Markusevich (Printice-Hall, 1965).
2. Introduction to the theory of entire function. A.S.B. Holland (Academic Press New York and London, 1973).
3. Functions of One Complex Variable.J. B. Conway, 2nd edition (Narosa Publishing House, New Delhi, 1997).

Reference Books:

1. Introduction to the Theory of Function of a Complex Variable. E. T. Copson (Oxford University press, 1970).
2. TheTheory of Functions. E. C. Titchmarsh, 2nd edition (Oxford University Press, 1970).
3. Complex Analysis. L. V. Ahlfors, 3rd edition (McGraw-Hill, 1979).
4. Lectures on the general theory of integral functions. G. Valiron (Toulouse, 1923).
5. Entire Functions. R. P. Boas (Academic Press, 1954).
6. Theory of Analytic Functions. H. Cartan (Dover Publication, 1995).

Course – MSMP305-4
Measure and Integration-I (Marks – 50)

Total Lectures Hours: 50H

Objectives

To present the concepts of measure and integration with respect to a measure.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. ring, σ-ring, algebra and σ-algebra of sets, monotone class of sets.

ii. measure and its basic properties, outer measure and measurability.
iii. measurable space, measure space, measurable functions, convergence in measure, convergence almost everywhere, convergence almost uniformly.
iv. Integration with respect to a measure.

Skills: The students would be able to
i. construct an arbitrary measure on a non empty set.
ii. determine measurable set and measurable function.
iii. calculate integral of a measurable function with respect to a measure.

General competence: The students would gain
i. general idea of measure, outer measure, measurability, measurable functions and integration with respect to an arbitrary measure, which will be useful for further studies in real analysis, probability theory.
ii. analytical and reasoning skills, which improve their thinking power.

Contents:

Ring, σ-ring, algebra and σ-algebra of sets, monotone class of sets, Borel sets, F_σ and G_δ sets. [6H]

Countably additive set function, measure and its basic properties, hereditary class, outer measure and measurability, extension of measures, complete measures and completion of a measure. [6H]

Construction of outer measures, regular outer measures. [8H]

Measurable functions, approximation of measurable functions by simple functions, Egoroff's theorem, Lusin's theorem, convergence in measure, convergence almost everywhere, convergence almost uniformly. [10H]

Integrals of simple functions, integral of measurable functions, properties of integrals and integrable functions, monotone convergence theorem, Fatou's lemma, dominated convergence theorem, Vitali convergence theorem. [20H]

Text Books:

1. Measure Theory. P. R. Halmos (Springer-Verlag, 1974).
2. Measure and Integration. S. K. Berberian (The Macmillan Co., 1965)
3. Measure Theory and Integration. G. D. Barra (New Age International (P) Ltd, 2013).

Reference Books:

1. Real and Abstract Analysis. E. Hewitt & K. Stormberg (John Wiley, 1965).
2. Real Analysis. H.L.Royden, 3rd edition (PHI, 2002).
3. Theory of Functions of a Real Variable, Vol. I & II. I. P. Natanson (Fedrick Unger Publi. Co., 1961).
4. Real and Complex Analysis. W. Rudin (Tata Mc Graw Hill, 1993).

5. Measure, Integration and Function Spaces.Charles Swartz, (World Scientific, 1994).
6. An Introduction to Measure and Integration.I. K. Rana (Narosa Publishing House, 1997).

Course: MSMP306-1

Euclidean and non – Euclidean Geometries-I (Marks-50)

Total Lectures Hours: 50H

Objectives

To present a systematic introduction of the fundamentals of the foundations of geometry, incidence geometry, Hilbert's plane, neutral geometry, affine geometry, projective geometry and Euclidean geometry.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. foundations of geometry.
ii. Euclid's postulates with their flaws.
iii. incidence geometry.
iv. categorical axiomatic system.
v. non Archimedean fields.
vi. Hilbert's plane.
vii. neutral geometry.
viii. affine geometry.
ix. projective geometry.
x. Euclidean geometry.

Skills: The students would be able to

i. apply projective and affine plane as model of incidence geometry.
ii. determine equivalence of Euclid's fifth postulate and Hilbert's parallel postulates.
iii. construct non-classical affine plane.
iv. classify conics in affine and projective planes.
v. establish the difference of a specific theorem in real Euclidean, affine and projective planes.
vi. acquire the applications of Euclidean, incidence, affine and projective geometries.

General competence: The students would gain

i. general idea of Euclidean and non-Euclidean geometries which will be useful for further studies.
ii. understanding about several equivalent statements of Euclid's fifth postulate.
iii. the notion of rational plane and surd plane.
iv. understanding about a particular result varies in different planes.
v. experience to construct non-Archimedean fieldusing mathematical tools.
vi. the concept of application of Euclidean, incidence, affine and projective geometries in mechanics, nature, architecture engineering, painting, medical instruments etc.

Contents:

Foundations of geometry: Ancient and modern point of view of geometry, axiomatic method, undefined terms, reductio ad absurdum proof, Euclid's postulates, the parallel postulate, attempts to prove the parallel postulate, equivalent statements of parallel postulate. [4H]

Incidence geometry:Incidence geometry, incidence axioms, models of incidence geometry, model isomorphism, model consistency, categorical axiomatic system, projective and affine plane as model of incidence geometry. [4H]

Hilbert's plane: Axioms of betweenness, axioms of congruence, existence of rigid motion, ordering of segments and angles, axioms of continuity (circle-circle, line-circle and segment-circle continuity principle, Archimedes axiom, Aristotle's axiom, Dedekind's axiom),Hilbert's axioms of parallelisms, Hilbert plane, real Euclidean plane/real Cartesian plane, Euclidean plane, rational plane/constructible Euclidean plane, Pythagorean plane, hyperbolic plane, Hilbert's field, isomorphism of Hilbert plane and real Cartesian plane (statement only), classification of Hilbert's plane (statement only), non-Archimedean field (existence and examples only). [6H]

Neutral geometry: Definition and examples of neutral geometry, Saccheri and Lambert quadrilaterals, semi-Euclidean plane, Saccheri's angle theorem, non-obtuse angle theorem and Saccheri-Legendre theorem (statement only), equivalence of Euclid's fifth postulate and Hilbert's parallel postulates. [4H]

Affine geometry:Definition and examples of affine plane,algebraic model of affine plane, order of an affine plane, classical affine plane, examples of non-classical affine plane, isomorphism types of affine planes up to order 31, parallelism and simple properties, combinatories of finite planes, affine planes over finite fields, affine transformations, collineations in affine plane, affine coordinates, triangles and parallelograms, statements of classical theorems in affine plane (Menelaus theorem, Ceva's theorem, Desargues theorem, Pascal's theorem). [10H]

Projective geometry:Definition and examples of projective planes, algebraic model of projective plane, order of aprojective plane, smallest projective plane, isomorphism types of projective planes up to order 31,finite projective planes, projective completion, homogeneous coordinates, projective transformations, collineations in projective plane, principle of duality, statements of classical theorems in projective plane (Desargues theorem, Pappus-Pascal theorem, Fano's theorem), projective line, projective completion of conics. [10H]

Classification of conics: Affine classification of conics, projective classification of conics, transitive groups on affine conics. [5H]

Euclidean Geometry: n-dimensional Euclidean space E^n, isometries of E^2, triangles and parallelograms, length minimizing curves in E^n, geometry of plane curves. [7H]

Text Books:

1. An Expedition to Geometry. S. Kumaresan & G. Santhanam (Hindustan Book Agency, 2005).
2. Euclidean and non-Euclidean Geometries: Development and History. M. J. Greenberg, 4th edition (W. H. Freeman and Company, New York, 2008).

Reference Books:

1. Incidence Geometry. G. E. Moorhouse (Univ. of Wyoming, Math5700-Fall2700, August, 2007).
2. A History of Mathematics. C. B. Boyer & U. Merzbach, 2nd edition (New York, Wiley, 1991).
3. What is Mathematics? R. Courant & H. Robbins (Oxford Univ. Press, New York, 1941).
4. Introduction to Geometry. H. S. M. Coxeter (New York, Wiley, 2001).
5. Thirteen Books of the Elements, 3 Vols. Tr. T. L. Heath, with annotations, Euclid (New York, Dover, 1956).
6. Introduction to non-Euclidean Geometry. H. E. Wolfe,(New York, Holt, Rinehart and Winston, 1945).

Course: MSMP306-2

Commutative Algebra I (Marks - 50)

Total lectures Hours: 50H

Objectives

To present a systematic introduction of the fundamentals course on commutative algebra I.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge on

i. finitely generated modules, basis of free modules, tensor product of two R-algebras, behaviour of tensor product with respect to exact sequence.
ii. localization of rings and their properties
iii. Noetherian modules and Artinian modules.

Skills: The students would be able to

i. compute basis of a free module and tensor product of two R-algebras.
ii. develop skills localization of rings
iii. apply going-up and going-down theorems in various problems

General competence: The students would gain

i. general idea of constructing basis of a free module
ii. understanding localization, projective R-module and integral extension.
iii. expertise in solving many tricky problems in commutative algebra

Contents:

(Throughout R is assumed to be a commutative ring with 1)

Modules: Cyclic modules, finitely generated modules, direct sum of submodules, free modules, basis of a free module, exact sequence, split exact sequence, the R-module $Hom_R(M,N)$ and its properties, where M,N are R-module, projective R-module-examples and properties, Shanuel's lemma, tensor product, existence and uniqueness of tensor product of two R-modules, scalar extension of an R-module, tensor product of two R-algebras, behaviors of tensor product with respect to exact sequences, flat R-module –examples and properties, faithfully flat R-module, flat and faithfully flat R-algebras. [15H]

Localization: Ideals, operations on ideals, co-maximal ideals and properties, nilpotent elements, nil radicals, Jacobson radicals, Chinese remainder theorem, extension and contraction of ideals – their properties, local rings, Nakayama's lemma, localization, localization of localization, rank of projective R-module, patching up of localization, applications. [10H]

Noetherian modules: Hilbert basis theorem, primary and irreducible submodule and related properties, primary decomposition, first and second uniqueness theorems, Artinian module, length of a module, a composition series of an R-module, Jordan-H$\ddot{o}$lder theorem, properties of Artinian modules. [10H]

Integral extension: Integral elements and its properties, integral closure, integrally closed set, integral extension, going-up theorem,integrally closed domain,going-down theorem, finiteness of integral closure, Noether's normalization theorem, weak Nulstellensatz theorem. [15H]

Text Books:

1. Commutative Algebra. N.S. Gopalakrishnan, 2nd edition (Orient Blackswan Private Limited)
2. Introduction to Commutative Algebra. M. Atiya & I.G. MacDonald, 1st edition (Indian) (Levant Books, India, 2007).

Reference Books:

1. Abstract Algebra. D.S. Dummit & R.M. Foote, 3rd edition (Wiley, 2016).

2. Module Theory: An Approach to Linear Algebra. T. S. Blyth, 1st edition(Clarendon Press, 1977).
3. Fundamentals of Abstract Algebra. D.S. Malik, J.N. Mordenson, M.K.Sen, 1st edition (Mcgraw-Hill Book Company – Koga, 1997)
4. Basic Algebra II. N. Jacobson, 1st edition (Hindusthan Publishing Corporation, India, 1984).
5. Commutative Algebra-with a View Toward Algebraic Geometry. D. Eisenbud, 1st edition (Springer-Verlag New York, 1995).

Course: MSMP306-3
Advanced Operator Theory –I (Marks-50)

Total Lectures Hours: 50H

Objectives: To present a systematic knowledge on operator theory in advance level with the introduction of spectral theory.

Learning outcomes: On completion of the course the students should have the following outcomes defined in terms of knowledge, skill and general competence.

Knowledge: The students will gain knowledge on

(i) compact operators, the basic ideas of spectral theory, weakly compact operators, operator equations involving compact operators.
(ii) various properties on compact and weakly compact operators, spectral properties of compact and normal operators.
(iii) compact extension and compact integral operators.
(iv) spectral representation of normal operators and theorems of Fredholm types.

Skills: The students will be able to

(i) compare the spectral properties of operators defined on infinite dimensional spaces to that of operators defined on finite dimensional spaces.
(ii) apply the results of compact operators for solving the problems related to integral operators.
(iii) find the spectrum and resolvent set of different kinds of operators like bounded linear, compact, normal, compact normal etc.
(iv) compare to the topological properties of spectrum of different kinds of operators.

General competence : The students will gain

(i) general ideas on the properties of compact operator and sequence of compact operators and weakly compact operators.
(ii) basic ideas on spectral theory of operators.
(iii) spectral properties of bounded linear, compact and normal operators.

Contents:

Basic ideas of spectral theory of linear operators on finite and on arbitrary dimensional normed linear spaces, eigen values, resolvent set, spectrum, continuous spectrum, residual spectrum, point spectrum, approximate spectrum. [4H]

Spectral properties of bounded linear operators on Banach spaces, spectral mapping theorem, use of complex analysis in spectral theory, local holomorphy and holomorphy of operator functions, holomorphy of resolvent operators, spectral radius of bounded linear operators. [8H]

Compact operators on normed linear spaces and theirs properties, uniform convergence of sequence of compact operators, adjoint and conjugate of compact operators, compact extension, compact integral operators, weakly compact operators and its properties. [10H]

Spectral properties of compact operators,eigen values, range and null spaces of compact operators, representation of a normed linear space as a direct sum of range space and null space. [10H]

Operator equations involving compact operators, theorems of Fredholm type, lemma of biorthogonal system, Fredholm alternative theorem. [10H]

Normal operators, spectral properties of normal operators, spectral representation theorem of bounded normalfinite dimensional operators. [8H]

Text books:

1. Functional Analysis. G. Bachman & L. Narici (Academic Press, 1966).
2. Introductory Functional Analysis with Applications. E.Kreyszig (Wiley Eastern, 1989).

Reference books:

1. Elements of Functional Analysis. B. K. Lahiri (The World Press Pvt. Ltd., Kolkata, 1994).
2.Functional Analysis. B. V. Limaye (Wiley Eastern Ltd, New Delhi, 1981).
3.Functional Analysis. M. T. Nair (Prentice-Hall of India Pvt. Ltd, New Delhi, 2002).
4.Functional Analysis. K. Yosida, 3rd edition (Springer Verlog, New York, 1990).

Course: MSMP306-4
Ergodic Theory-I(Marks-50)

Total Lectures Hours: 50H

Objectives

To present a systematic introduction of the fundamentals course on ergodic theory I.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. Application of measure theory, especially Haar measure on locally compact topological groups on ergodic theory.
ii. Importance of Pόincare recurrence theorem and its application to various interesting problems on $\mathbb{R}$.
iii. Isomorphism, conjugacy and spectral isomorphism.
iv. Measure preserving transformation with discrete spectrum.
v. Entropy and related problems.

Skills: The students would be able to

i. Implement Pόincare recurrence theorem in different problems of ergodic theory.
ii. To investigate whether transformations like Markov shift or rotation on a compact topological group are ergodic.
iii. Verify whether a measure preserving transformation is isomoprphism and to study its consequences.
iv. To analyze how good an entropy is.

General competence: The students would gain

i. General idea about the importance ergodic theory
ii. To analyze common features of different concepts in ergodic theory
iii. Expertized in solving many tricky problems in ergodic theory.

Contents:

Prerequisites: Measure spaces, integration, absolutely continuous measure and conditional expectations, function spaces, topological groups, Haar measure, character theory, endomorphism of tori, Perron-Frobenius theory. [10H]

Measure-preserving transformation: Definitions and examples, problems in ergodic theory, associated isometries, recurrence, ergodicity, ergodic theorem, mixing**.** [10H]

Isomorphism, conjugacy, and spectral isomorphism, point maps and set maps, isomorphism of measure-preserving transformations, conjugacy of measure preserving transformations, the isomorphism problem, spectral isomorphism, spectral invariants. [10H]

Measure preserving transformation with discrete spectrum, eigenvalues and eigen functions, discrete spectrum, group rotations. [5H]

Entropy, partition and subalgebras, entropy of a partition, conditional entropy, entropy of a measure preserving transformation, properties of $h(T, \mathcal{A})$ and $h(T)$, invariance of entropy, Bernouli automorphisms and Kolmogorov automorphisms, Pinsker $\sigma -$algebra of a measure preserving transformation, sequence entropy, non-invertible transformations. [15H]

Text books:

1. An Introduction to Ergodic Theory. P. Walters, 1st edition (Springer, 2004).

Reference books:

1. Ergodic Theory and Information. Patrick Billingsley, 1st edition (Robert E. Krieger Publishing Co., 1978).
2. Basic Ergodic Theory. M. G. Nadkarni, TRIM 6 (Hindustan Book Agency, 1995).
3. Recurrence in ergodic theory and combinatorial number theory. H. Furstenberg, 1st edition (Princeton University Press 1981).
4. Ergodic theory. K. Petersen, Cambridge Studies in Advanced Mathematics (2) (Cambridge University Press, 1989).
5. Ergodic theory with a view towards number theory. M. Einsiedler and T. Ward, 1st edition (Springer-Verlag London, 2011).

MSMP306-5
Algebraic Topology-I(Marks - 50)

Total lectures Hours: 50H

Objectives

To present a systematic introduction of the fundamentals course on algebraic topology I.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. the importance of category theory in algebraic topology and the category theory independently as a process of unification.
ii. homotopy theory and construction of fundamental groups of a path connected space.
iii. a brief coverage on universal cover and its existence.

Skills: The students would be able to

i. calculate fundamental groups using covering space.
ii. analyze homomorphisms and automorphisms of covering spaces.
iii. apply Borsuk-Ulam theorem for S^2 and Brower fixed-point theorem

General competence: The students would gain

i. general ideas of development of category theory from the functorial correspondence between the path connected spaces and its fundamental groups.
ii. to compute lifting of paths to a covering spaces.
iii. expertise in solving many tricky problems in algebraic topology

Contents:

Category: Definitions and some examples of categories, functors and natural transformations. Universal arrows, Yoneda's lemma, coproduct and colimits, products and limits, representable functors. [10H]

Homotopy: Definition and some examples of homotopies, homotopy type and homotopy equivalent spaces, retraction and deformation, H-space. [8H]

Fundamental group and covering spaces: Definition of the fundamental group of a space, the effect of a continuous mapping on the fundamental group, fundamental group of a product space, notion of covering spaces, lifting of paths to a covering space, fundamental groups of a circle. [15H]

Universal cover, its existence, calculation of fundamental groups using covering space, projection space and torus, homomorphisms and automorphisms of covering spaces, Deck transformation group, Borsuk – Ulam theorem for S^2, Brower fixed-point theorem in dimension 2. [17H]

Text Books:

1. Algebraic Topology.A. Hatcher, 1st edition (Cambridge University Press, 2003).
2. Topology, J. Munkres, 2nd edition (Pearson, 2015)
3. Category Theory for working mathematician. S. Maclane, 2nd edition (Springer-Verlag New York, 1998)

Reference Books:

1. Basic Concepts of Algebraic Topology. F. H. Croom, 1st edition (Springer, NY, 1978.)
2. Algebraic Topology. E. H. Spanier, 1st edition (McGraw-Hill,1966)
3. A Basic Course in Algebraic Topology. W. S. Massey (Springer-Verlag, New York Inc., 1991)
4. Lecture Notes on Elementary Topology and Geometry. I. M. Singer & J. A. Thorpe, 1st edition (Springer, India 2003)

Course: MSMP307
Community Engagement Activities (Marks - 25)

Each student is to carry out some work related to the development/welfare of a community/society as per the guidelines of the Department to be framed from time to time. Marks of the Internal Assessment will be awarded through viva-voce and marks of the term-end examination will be awarded through the evaluation of the report to be submitted by the respective student.

Course: MSMA401
Dynamical Systems, Chaos and Fractals (Marks 50)
Total lectures Hours: 50H

Objectives

To study non-linear dynamics, bifurcation theory, chaos and fractals.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. non-linear dynamics and bifurcation theory.
ii. chaos theory.

Skills: The students would be able to

i. apply the knowledge in non-linear dynamics.
ii. quantify the chaotic motions and its connects with fractals.

General competence: The students would gain

i. general idea of non-linear dynamics, bifurcation theory and chaos.
ii. fractals with applications

Group A: Dynamical Systems (Marks: 25)

Dynamical systems: Dynamical systems, a brief history, Continuous and discrete systems, Flows and maps, fixed points, stability, Qualitative analysis of one-dimensional flows, periodic points and their stabilities, Different attractors, $\omega-$ and $\alpha-$ limit points, wondering point, Basin of attraction. [5H]

Stability theory of fixed points: Hyperbolic fixed point, hyperbolicity, Stable, unstable and center subspaces, Hyperbolic flow, Invariant manifold, Stable manifold and center manifold theorems (statement only), Lyapunov stability, Local and global stabilities, periodic orbits, limit cycles, attracting and invariant sets, Poincare-Bendixson theorem, Poincare map, Lienard's theorem (statement only) and applications. [8H]

Bifurcations: Saddle-Node, Pitch-Fork and Transcritical bifurcations for one-dimensional continuous systems, period-doubling and flip bifurcations, Hopf-bifurcation, Quasi-periodic and Neimark-Sacker bifurcations, Analysis of Lorentz systems. [7H]

Some important maps: Tent map, Logistic map, Baker map, Shift maps, Gaussian map, Henon map and their properties. [5H]

Group B: Chaos and Fractals (Marks: 25)

Maps: Topological conjugacy and semi-conjugacy among maps, Properties, Applications. [3H]

Chaotic maps: Topological transivity, Sensitive dependence on initial conditions (SDIC) property, Mixing, Chaotic map, Definition of logistic map, Periodic windows, Boundary of chaos, The Sawtooth map, Poincare map, Circle and Horse-Shoe maps.

Quantification of Chaos: Universal sequence, Feidenbaum number, Lyapunov exponent and Invariant measure, Sharkovskii order and theorem (statement only), period 3 implies chaos, Routes of chaos and Universality in chaos. [12H]

Fractals: Basic ideas and properties, Self-similarity, Self-similar fractals, von-Koch curve, Sierpinsui triangle, Cantor set, Cantor dust, Mandelbrot set, Dimensions of fractals, Similar, Box and Housdorff dimensions, Pointwise and correlation dimensions, Strange attractors, Multifractals. [10H]

Text Books:

1. Nonlinear Dynamical and Chaos. Steven H. Strogatz (Perseus Books, Indian Edition, 2007).
2. An Introduction to Dynamical Systems and Chaos. G. C. Layek (Springer, 2015).
3. Chaos and Nonlinear Dynamics. Robert C. Hilborn (Oxford University Press, 2001).

Reference Books:

1. An Introduction to Chaotic Dynamical Systems. R. L. Devaney (Addition-Wesley, 1989).
2. Fractals Geometry, Mathematical Foundations and Applications. K. Falconer (Wiley, New York, 1990).
3. Chaos: An Introduction to Dynamical Systems. K.T. Alligood, T. D. Souer & J. A. Yorke (Springer, 1997).

Course: MSMA402

Fluid Mechanics (Marks: 50)

Total lectures Hours: 50H

Objectives

To expose the learners to the basics and principles of fluid mechanics.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. rotational and irrotational flows, velocity potential, stream function, source, sink, doublet etc.
ii. inviscid, viscous fluids and their governing equations.
iii. two dimensional, vortex motion of fluids.

Skills: The students would be able to

i. obtain dimensionless and vector invariant forms of governing equations of inviscid and viscous fluid flows
ii. find exact solutions of some specific problems of viscous fluid flows.
iii. determine flow features of inviscid and viscous fluid.
iv. determine the drag and lift forces for viscous fluid flows.

General competence: The students would gain

i. general idea of fluid mechanics which will be useful for further studies in specialized area viz. aerodynamics, thermodynamics etc.
ii. understanding about fundamental equations of flow and energy.

experience to construct mathematical models to tackle complex fluid flow problems.

Contents:

Basic concepts of fluid: Continuum hypothesis, Fluid and its classification, Newton's law of viscosity, Classification of fluid motion. [4H]

Inviscid fluid: Inviscid incompressible fluid, Constitutive equation, Euler's equation of motion & its vector invariant form, Bernoulli's equation and applications to some special cases, Helmholtz's equation for vorticity, Impulsive generation of motion and some properties, Boundary Conditions. [10H]

Irrotational motion of fluid: Irrotational motion, Velocity potential, Circulation, Kelvin's circulation theorem, Irrotational motion in simply connected and multiply connected regions, Kelvin's theorem of minimum kinetic energy, Acyclic irrotational motion and some properties (Using Green's theorem), Stream function, Complex potential, Two dimensional motion, Source, sink and doublets for two dimensional flows, Complex potentials for simple source, sink and doublet, Circle theorem, Uniform flow past a circle, Image of a source with respect to a plane boundary, image of a source outside a circle, image of a doublet outside a circle, Motion of translation and rotation of circular cylinder in an infinite liquid, Blasius theorem, Kutta-Joukowski's theorem, Axi-symmetric motion, Stokes' stream function, Three-dimensional motion, Source, sink, doublet for three dimensional flows. [16H]

Vortex motion: Vortex lines, vortex filaments and vortex surface, Helmholtz's theorems for vorticity, System of vortices, Rectilinear vortices, Vortex pair and doublets, Image of vortex with respect to a circle. [8H]

Flow of viscous incompressible fluid: Viscous incompressible fluid, Navier-Stokes' Equations of motion, Boundary conditions, Reynolds number and its significance, Poisueile flow, Couette flow, Flow through parallel plates, Flow through pipes of circular and elliptic cross sections,Vorticity transport equation, Energy dissipation due to viscosity. [12H]

Text Books:

1. An Introduction to Fluid Dynamics. G. K. Batchelor (Cambridge University Press, 2005).
2. Fluid Mechanics. P. K. Kundu & I. M. Cohen, 4thedition(Academic Press, 2008).
3. Text Book of Fluid Dynamics. F. Chorlton (Van Nostrand Reinhold Co., London, 1990)

Reference Books:

1. Foundations of Fluid Mechanics. S.W. Yuan, (PHI, 1970).
2. Viscous Fluid Dynamics. J. L. Bansal (Oxford and IBH Publishing Co., 1977).
3. Physical Fluid Dynamics. D.J. Tritton, 2nd edition(Oxford Science Publications, 1988).

Course: MSMA403
Introduction to Quantum Mechanics and Wavelet Analysis(Marks: 50)

Total lectures Hours: 50H

Objectives

To make the students familiar with the elementary ideas of quantum mechanics and wavelets.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. mathematical formalisms of quantum mechanics.
ii. Harmonic oscillator.
iii. conservation laws, symmetries, angular momentum and spin.
iv. structural properties of one-electron atoms.
v. approximate solutions of the time-independent Schrodinger equation.
vi. Square integrable functions
vii. Wavelet and wavelet transform'
viii. Multiresolution analysis

Skills: The students would be able to

i. apply the principles of quantum mechanics to calculate some important observables for a given wave function.
ii. solve the time-dependent Schrodinger equation for some model potentials.
iii. combine spin and angular momenta.
iv. determine bound state energies and wave functions approximately for some model potentials and two-electron atoms/ions.
v. Use Wavelet transform and changes through that.

General competence: The students would gain

i. general idea of non-relativistic quantum mechanics which will be useful for further studies in theoretical physics.
ii. understanding about fundamental quantum mechanical processes in nature.
iii. experience to construct approximate quantum mechanical models using mathematical tools
iv. Knowledge of Fourier transform in a new background.
v. Idea of wavelets

Contents:

Group A
Introduction to Quantum Mechanics (30 marks)

Origins of quantum theory: Inadequacies of classical mechanics; Planck's quantum hypothesis; Photoelectric effect; Compton experiment; Bohr model of hydrogenic atoms, Wilson-Sommerfeld quantization rule, Correspondence principle, Stern-Gerlach experiment (brief description and conclusion only). [6H]
Wave aspect of matter: de Broglie hypothesis; mater waves; uncertainty principle; double-slit experiment; Concept of wave function; Gedanken experiments. [5H]
Schrodinger equation: Time-dependent Schrodinger equation; Statistical interpretation – conservation of probability, equation of continuity, expectation value, Ehrenfest theorem; Formal solution of Schrodinger equation – time-independent Schrodinger equation, stationary state, discrete and continuous spectra, parity. [5H]
Solutions of Schrodinger equation in one-dimension: Infinite potential box; Step potential; Potential barrier; Potential well. [4H]
Linear harmonic oscillator in one-dimension: Classical description; Schrodinger method of solution; Energy levels and wave functions; Planck's law. [4H]
Hydrogenic atoms: Schrodinger equation for hydrogenic atoms; Solution in spherical polar coordinates; Spherical Harmonics, Energy levels and wave functions; Radial probability density. [3H]
Mathematical foundations of quantum mechanics: Concept of wave function space and state space; Observables; Postulates of quantum mechanics; Physical interpretations of the postulates – expectation values, Ehrenfest theorem, uncertainty principle. [3H]

Group B

Wavelet Analysis (Marks - 20)

Introduction, Review of Lp -spaces and Fourier transforms; Orthonormal bases, Riesz bases, Continuous and discrete wavelet transforms with basic properties, Orthonormal wavelets,Haar wavelets. [8H]

Multiresolution analysis (MRA), Bandlimited functions. [7H]

Applications to wavelet transform for physical systems. [5H]

Test Books:

1. Introduction to Quantum Mechanics. D. J. Griffiths (Pearson Prentics Hall, Upper Saddle River, NJ, 2005).
2. Quantum Mechanics. S. N. Ghoshal (S Chand & Company Ltd, Kolkata, 2002).
3. An Introduction to Wavelet Analysis. David F. Walnut (Birkhauser, 2008).

Reference Books:

1. Quantum Mechanics. B. H. Bransden and C. J. Joachain (Prentics Hall, 2005).
2. Quantum Mechanics, L. I. Schiff (McGraw-Hill, 1968).
3. Wavelet Transforms and Their Applications. L. Debnath (Birkhauser Boston, 2002).
4. Introduction to Wavelets and Wavelet Transforms. C. Sidney Burrus, Ramesh A. Gopinath & Haitao Guo (PHI,1998).

Course: MSMA404-1

Boundary Layer Theory and Magneto-hydrodynamics – II (Marks: 50)

Total lectures Hours: 50H

Objectives

To expose the learners to the fundamentals of Magneto-hydrodynamics

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge on

i. mathematical formalisms of magnetohydrodynamics.
ii. electromagnetic induction, magnetohydrostatics, force free field etc.
iii. dynamo theory.

Skills: The students would be able to

i. describe the motion of electrically conducting fluid under the applications of external magnetic force.
ii. solve the problems of magnetohydrostatics.
iii. solve some physical phenomena of magnetohydrodynamics.

General competence: The students would gain

i. general idea on steady/unsteady flow behaviour of electrically conducting fluid under the application of external magnetic field
ii. understanding about fundamental mechanical processes in presence of magnetic field. experience on the applicability of magnetohydrodynamics.

Contents:

Electrodynamics: Laws of electrodynamics, Electro-magnetic fields, Electromagnetic induction, Faraday's law, Energy in magnetic field, Maxwell's equations, Reminder of electrodynamics laws before Maxwell, Physical significance of Maxwell's equations, Related boundary conditions, Energy transfer and Poynting theorem. [6H]

Magnetohydrodynamics: What is MHD? A brief history of MHD, Physical description of electrically conducting fluids, Maxwell's electromagnetic field equations, Basic MHD equations, Energy flow, Lorentz force, Ohm's law. [7H]

MHD approximations: The low frequency dynamics of the electromagnetic field, Conservation laws for mass, momentum and energy in MHD, Dimensional analysis and Lundquist criterion, Convection dominated flows. [9H]

MHD wave propagation: The full significance of Faraday's law, Faraday's law in ideal conductor, Alfven's theorem and its physical interpretation, Flux-freezing phenomena, Diffusion dominated case, Physical interpretation of Lorentz force, Alfven waves. [7H]

Steady MHD flow: Parallel steady flow, One-dimensional steady viscous flow, Hartmann flow, Couette flow. [6H]

Unsteady MHD flow: MHD Rayleigh problem. [4H]

Magnetohydrostatics: Basic equations, Pinch effect, Linear pinch, Stability of pinch configurations with applications. [4H]

Force free-field: Basic equations and their general solutions, Toroidal and Poloidal fields. [3H]

Geo-magnetism: Dynamo problem in earth, Dynamo theory, Symmetric fields, Cowling's theorem, Isorotation, Ferraro's law of isorotation. [4H]

Text Books:

1. Magnetohydrodynamics. T.G. Cowling (Intersicence Publishers Ltd., 1956).
2. An introduction to Magneto-Fluid Mechanics. V.C.A. Ferraro & C. Plumpton (Clarendon Press, 1966).

Reference Books:

1. A text book of Magnetohydrodynamics. J.A. Shercliff (Pergaman Press, 1965).

2. An Introduction to Magnetohydrodynamics. P. A. Davidson (Cambridge University Press, 2001).

Course: MSMA404-2

Turbulent Flows – II (Marks 50)

Total lectures Hours: 50H

Objectives

To developtwo-point statistics systematically in turbulence, Kolmogorov and non-Kolmogorov turbulent flows.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. Kolmogorov hypotheses and their limitations.
ii. scaling laws in turbulence.

Skills: The students would be able to

i. apply the knowledge of turbulence in different fields of practical interest.
ii. statistical analysis of random motions.

General competence: The students would gain

i. general idea of turbulent flows, scaling laws.
ii. non-equilibrium turbulence

Contents:

Two-point statistics: Double correlation between velocity components, Triple and multi correlation functions, Longitudinal and lateral correlations, Correlations in homogeneous and isotropic turbulence, Self-similarity, Karman-Howarth equation, Decay of isotropic turbulence. [12H]

Scales of turbulence: Turbulence as a multi-length scales phenomenon,Integral, lateral, longitudinal and Taylor length scales and their relations. [5H]

Invariant in turbulence: Concept of invariant in turbulent flows,Loitsyansky, Birkhoff and Saffman invariants. [5H]

Kolmogorov turbulence: Richardson–Kolmogorov cascading, Energy cascade, Spectral analysis of turbulence, Energy spectrum, Kolmogorov's equilibrium hypothesis, Kolmogorov's scalings, Kolmogorov's hypotheses, Structure functions, Kolmogorov two-third, four-fifth laws, Effect of a finite viscosity, The dissipation range, The Richardson cascade and the localness of interactions, Kolmogorov and Landau: the lack of universality, Non-equilibrium turbulence. [12H]

Turbulence Intermittency: Self-similar and intermittent random functions, Log-normal model, Novikov-Stewart Model, Mandelbrot Random curdling model, The β-(beta) model, The multifractal model. [5H]

Turbulent boundary layer flows: Description of turbulent boundary layer flows, Mean-momentum equations, Two-layer hypothesis, Overlap region, Logarithmic law, Channel flow and Couette flow. [6H]

Turbulence models: Eddy viscosity and mixing length models and their limitations, One equation and two-equations models, Remarks on turbulence modelling. [5H]

Text Books:

1. Turbulance, the Legacy of A. N. Kolmogorov. U. Frisch (Cambridge University Press, 1995).
2. Turbulent Flows. S. B. Pope (Cambridge University Press, 2000).
3. Fluid Mechanics. P. K. Kundu (Academic Press, 1990).

Reference Books:

1. The Theory of Homogeneous Turbulence. G. K. Batchelor (Cambridge University press, 1953).
2. Turbulence. J.O. Hinze, 2nd edition (McGraw-Hill, New York, 1977).
3. Physical Fluid Dynamics. D. J. Tritton, 2nd edition (Oxford Science Publications, 1988).

Course: MSMA404-3

Space Sciences – II (Marks 50)

Total lectures Hours: 50H

Objectives

Basic astronomy is studied in this course. Types of telescopes, merits and demerits etc will be studied. Structure and constituents of galaxies are studied. Cosmological redshifts will be studied. We will know that our universe is passing through a late time cosmic acceleration. We will study the nooks and corners of it.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. mathematical formalisms of cosmology.
ii. Galaxies
iii. Hubble’s law.
iv. Late time cosmic acceleration, dark energy.

v. Missing mass, dark matter.

Skills: The students would be able to

i. apply the fundamental laws of cosmology solve the time-dependent Schrodinger equation for some model potentials.
ii. Construct Friedmann equations.
iii. Know about galaxies, their precession speeds and constituents.

General competence: The students would gain

i. general idea of Cosmology
ii. understanding about fundamental structure, evolution of galaxies.
iii. Evolving states of universe.

Contents:

Introductions To Astronomy and Astrophysics: Sky coordinates and motions: Earth Rotation , Sky coordinates, seasons - phases of the Moon, the Moon's orbit and eclipses, timekeeping (sidereal vs synodic period); Planetary motions, Kepler's Laws, Gravity; Light & Energy, Telescopes, Optics - Detectors; Planets: Formation of Solar System, planet types, planet atmospheres - extrasolar planets; Stars: Measuring stellar characteristics (temperature, distance, luminosity, mass, size), HR diagram, stellar structure (equilibrium, nuclear reactions, energy transport) - stellar evolution; Galaxies: Our Milky Way, Galactic structure, Galactic rotation, Galaxy types, Galaxy formation; Cosmology: Expansion of the Universe - redshifts - supernovae, the Big Bang - history of the Universe - fate of the Universe.[9H]

Telescopes and Detectors: optical, infrared, radio, x-rays, gamma-rays, neutrinos and cosmic rays; Gravitational radiation; Detection of dark matter and Dark Energy Astronomy from Space. [5H]

Exoplanets: Trans-Neptunian Objects, Centaurs, Planetary rings, Planet formation: Evolution of protoplanetary disks, Growth of solid bodies, Formation of Terrestrial and Giant planets, Planetary Migration, Extrasolar Planets: Detection techniques, Estimating planetary masses, sizes, orbital parameters, Habitable zones: factors influencing habitable zone, continuously habitable zone, Missions to study Planets and Extrasolar planets: Overview and Results. [9H]

Structure and Evolution of Stars: Mechanical, Thermal and Nuclear time scales, Hydrostatic equilibrium (Newtonian and Relativistic), Polytropic Equation of State, Lane Emden Equation, Degenerate matter Equation of State, White Dwarfs and Chandrasekhar limit, Virial Theorem, Radiative Equilibrium, Schwarzschild convection criterion, nuclear energy generation, stages of nuclear burning, full set of stellar structure equations, example solutions, HR diagram and the main sequence, Schonberg-Chandrasekhar limit, post-main sequence evolution, Hayashi tracks, Horizontal branch, giant and asymptotic giant branches, planetary nebula formation, supernovae, compact objects. [9H]

Galaxies (Structure, Dynamics and Evolution): Classification of galaxies, contents and dimensions, collisionless stellar dynamics, relaxation time, dynamical friction, violent relaxation, galactic potential and orbits, spiral density wave and Lindblad resonance, rotation curves, Tully-Fisher relation, Central Black Holes and fundamental plane relationship, Mass and Luminosity function, Press Schechter formalism, Star formation history and chemical evolution, active galaxies and activity duty cycle, galaxies at high redshift, clusters and groups, evidence of dark matter. [9H]

Accretion Physics: Introduction: Accretion as a source of energy, observational consequences. Accretion in binary system: Introduction, Interacting binary system, Roche lobe overflow, Disk formation, Viscous torque,The α disk viscosity, Low and high-mass X-ray binaries. Accretion disk (thin accretion disk) Theory: Basic concepts, Structure of thin disk, The emitted spectrum of steady α-disk, Time dependence and stability, the thermal disk instability model (dwarf novae), wind accretion, Disk around young stars, confrontation with observations. Accretion onto compact object: Boundary layers, Accretion on to magnetized neutron star and white dwarf, accretion column, accretion on to black hole. Accretion disk in AGN: AGN models, Radio, millimeter and infrared emission, optical, UV and X-ray emission, broad and narrow line region, Extended and compact radio sources, The Blandford-Znajek model. Thick discs: The limiting luminosity, accretion tori, self-gravitating disks and their stability, astrophysical implication. Accretion flows: The governing equations, A unified description of steady flow, advection-dominated flows, general transonic accretion solution in presence of heating and cooling. [9H]

Text Books:

1. An Introduction to Modern Astrophysics. B W Carroll & D A Ostlie, 3rd edition (Addison-Wesley 1996).
2. The Physical Universe. Frank Shu, 2nd edition (University Science Books 1982).

Reference Books:

1. Astrophysical Concepts. Martin Harwit, 2nd edition (Springer, 1988).
2. Invitation to Astrophysics. T. Padmanabhan, 3rd edition (World Scientific Publishing Co, 2006).
3. Theoretical Astrophysics vols 1-3. T. Padmanabhan (Cambridge University Press., 2012).
4. Accretion Power in Astrophysics , Juhan Frank, Andrew King & Derek Raine (Cambridge University Press, 2002)
5. An introduction to the theory of stellar structure and evolution. Dina Prialnik, 2nd edition (Cambridge University Press, 2009)
6. High Energy Astrophysics vol 1. M.Longair (Cambridge University Press. 1987).

Course: MSMA405-1

Advanced Optimization -II (Marks - 50)

Total lectures Hours: 50H

Objectives

To present a systematic introduction of the fundamentals of advanced optimization techniques.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

(i) dynamic programming and its applications.

(ii) calculus of variations, performance indices, bang-bang control, bang-bang principle, Pontryagin's Maximum Principle.

(iii) Geometric programming with posynomial type objective and constraint functions

(iv) genetic algorithms & particle swarm optimization for solving nonlinear optimization problems

Skills: The students would be able to

(i) apply the Bellmen's principle for solving investment, routing, production-inventory, reliability optimization, L.P.P., cargo loading problems.

(ii) apply geometric programming for optimization problems with posynomial type objective and constraint functions.

(iii) solve different types of control problems.

(iv) develop metaheuristic algorithms for solving various types of optimization problems.

(v) perform research works for solving decision making problems in the areas of management science and engineering design.

General competence: The students would gain

(i) the general idea of advanced optimization techniques which will be useful for solving decision making problems.

(ii) the knowledge and understanding about the derivative free optimization techniques and how to solve the highly nonlinear and non-differentiable optimization problems.

(iii) the ability to solve the optimization problem with the help of optimal control theory.

Contents:

Dynamic programming: Basic features of dynamic programming problems, Bellman's principle of optimality, multistage decision process-forward and backward recursive approaches, Dynamic programming approach for solving (i) linear and non-linear programming problems, (ii) routing problem, (iii) reliability optimization problem, (iv) inventory control problem, (v) cargo loading problem (vi) Allocation problem. [16H]

Geometric Programming: Unconstrained and constrained geometric programming.[8H]

Optimal Control Theory: Introduction to Optimal Control: Control and Optimal Control, Examples, The Basic Optimal Control Problem, Variational Calculus; Optimal Control with Unbounded Continuous Controls, The Hamiltonian, Extension to Higher Order Systems; Bang-Bang Control, Pontryagin's Principle, Switching Curves, Transversality Conditions; Applications of Optimal Control in Economic Growth. [12H]

Computational optimization: Fundamentals of Genetic Algorithm & Particle Swarm optimization. [14H]

Text Books:

1. Optimization Techniques. C. Mohan & K. Deep (New Age Science, 2009).
2. Engineering Optimization Theory and Practice. Singiresu S Rao, 5th edition (John Wiley & Sons, Inc., 2020).

Reference Books:

1. Genetic algorithms + Data structures = evolution Programs. Z. Michalawich, 3rd edition (Springer Verlag, 1996).
2. Particle Swarm Optimization: Theory, Techniques and Applications. Andrea E. Olsson (Nova Science Publishers, 2011).
3. Introduction to Optimization Techniques: Fundamentals and Applications of Nonlinear Programming. M. Aokie (The Macmillan Company, 1971).
4. Operations Research: Theory and Applications, J. K. Sharma (Macmillan, 1997).
5. Optimal Control Theory: An Introduction. Donald E. Kirk (Dover Publications, 2012)·

Course: MSMA405-2
Advanced Operations Research-II (Marks 50)

Total lectures Hours: 50H

Objectives

To impart the knowledge of Operations Research so as to identify the most effective and efficient solutions to problems

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. different types of queueing systems.
ii. dynamic programming technique concepts and its applicability in decision making problems.
iii. concepts of reliability and system reliability.
iv. system replacement strategies.
v. optimal control theory

Skills: The students would be able to

i. formulate queueing models to analyse real world systems.
ii. solve the optimization problem using dynamic programming technique.

iii. apply reliability theory in different engineering model.
iv. deal the optimal control problem in real world.

General competence: The students would gain

i. general idea of optimal control theory which will be useful for further studies in nonlinear and multilevel problems.
ii. experience to construct queueing theory models for real life problem
iii. idea of the optimal system design through reliability theory
iv. experience to deal with reliability problem in practical

Contents:

Queuing theory: Non-Poisson queuing models-M/ E_k/1, M/G/1, Machine repairing problem, power supply model, cost models in queuing system, mixed queuing model M/D/1. [10H]
Dynamic Programming: Basic features of dynamic programming problems, Bellman's principle of optimality, multistage decision process-forward and backward recursive approaches, Dynamic programming approach for solving (i) linear and non-linear programming problems, (ii) routing problem, (iii) reliability optimization problem, (iv) inventory control problem, (v) cargo loading problem (vi) Allocation problem. [13H]
Optimal Control Theory: Introduction to Optimal Control: Basic Concepts and Definitions, Formulation of Simple Control Models, Variational Calculus; The Optimal Control Problem: The Mathematical Model, Constraints, Objective Function; Bang-Bang Control; Maximum Principle: The Hamilton-Jacobi-Bellman Equation, Derivation of the Adjoint Equation, Pontryagin's Maximum Principle; Applications of Optimal Control to Production and Inventory, Economic Growth, Exploited Populations, and Advertising Policies. [12H]
Reliability: Definition of reliability, Measures of reliability, system reliability, system failure rate, reliability of different systems, like series, parallel, series parallel, parallel-series, k-out-of-n, etc., idea of reliability optimization. [8H]
Replacement: Replacement problem, Types of replacement problems, Replacement of capital equipment that varies with time, Replacement policy for items where maintenance cost increases with time and money value is not considered, Money value, Present worth factor, Discount rate, Replacement policy for item whose maintenance cost increases with time and money value changes at a constant rate, Choice of best machine, Replacement of low cost items, Group replacement, Individual replacement policy, Mortality theorem, Recruitment and promotional problems. [7H]
Text Books:

1. Operations Research: Theory, Methods and Applications. S. D. Sharma & H. Sharma (Kedar Nath Ram Nath, 2012).
2. Reliability Engineering. K. C. Kapur, 1st edition (Wiley, 2014).
3. Optimal Control Theory: Applications to Management Science and Economics. S. P. Sethi& G. L. Thompson, 2nd edition (Springer, 2006).

Reference Books:

1. Operations Research – An Introduction. H. A. Taha, 10th edition (Pearson, 2017).

2. Introduction to Mathematical Control Theory. S. Barnett, 2nd edition (Oxford University Press, 1985).
3. Advanced Optimization and Operations Research. A. K. Bhunia, L. Sahoo & A. A. Shaikh (Springer, 2019).
4. Reliability Engineering. K. K. Aggarwal, 1st edition (Springer, 1993).
5. Introduction to Operations Research. F. S. Hillier, G. J. Lieberman, 7th edition (McGraw-Hill, 2001).
6. Operations Research: Theory and Applications. J. K. Sharma, 3rd edition (Macmillan, 2006).

Course: MSMA405-3

Quantum Mechanics-II(Marks 50)

Total lectures Hours: 50H

Objectives

i. To present a systematic introduction of the fundamentals of relativistic quantum mechanics.
ii. To present the theory of potential scattering.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. mathematical formalisms of relativistic quantum mechanics.
ii. theory of potential.
iii. quantum statistics.
iv. approximate solutions of the time-dependent Schrodinger equation.

Skills: The students would be able to

i. apply the principles of relativistic quantum mechanics to calculate some important observables for a given wave function.
ii. solve the time-dependent Schrodinger equation for calculating transitional probabilities.
iii. determine scattering parameters approximately for potential scattering.

General competence: The students would gain

i. general idea of relativistic quantum mechanics which will be useful for further studies in theoretical physics.
ii. understanding about fundamental quantum scattering processes in nature.

iii. experience to construct approximate quantum mechanical models using mathematical tools

Contents:

Relativistic quantum mechanics: Klein-Gordon equation – plane wave solution, interpretation of K-G equation; Dirac equation – covariant form, charged particle in electromagnetic field, equation of continuity, plane wave solution; Dirac hole theory; Spin of the Dirac particle. [7H]

Scattering theory: Basic concepts – types of scattering, channels, thresholds, cross sections; Classical description – equation of trajectory, cross sections, Hard-sphere scattering, Rutherford scattering; Quantum description – cross sections, Laboratory frame and centre of mass frame, optical theorem. [5H]

Method of partial waves for potential scattering: Description of the method; Phase shift; Convergence of partial wave series; Zero-energy scattering - scattering length, S-matrix, K-matrix, T-matrix; Relation between phase shift and potential; relation to cross sections – optical theorem. [7H]

Integral equation of potential scattering: Description of the method; Lippmann-Schwinger equation; Integral representation of scattering amplitude. [5H]

Scattering by Coulomb potential: Scattering state solution in parabolic coordinates; Cross sections; Modified Coulomb potentials. [4H]

Approximate methods for potential scattering: Born series – first and second Born amplitudes; Validity of FBA; eikonal approximation – description, scattering amplitude, cross sections WKB approximation: WKB method - connection formula; Validity; α - emission; Bound state in a potential well. [6H]

Variational methods in potential scattering: Differential form – Kohn variational method, inverse Kohn variational method, Hulhen variational method, Kohn-Hulthen variational method; Schwinger variational principle – scattering amplitude, phase shift, bound principle. [5H]

Quantum statistics: Fundamental assumption; Most probable configuration; Maxwell-Boltzmann distribution, Fermi-Dirac distribution and Bose-Einstein distribution; Black body spectrum. [5H]

Time-dependent perturbation theory: first-order perturbation; harmonic perturbation; transitions to continuum states; absorption and emission; Einstein's coefficients; Fermi's golden rule; selection rules; Rayleigh scattering; Raman scattering. [6H]

Text Books:

1. Quantum Mechanics. B. H. Bransden & C. J. Joachain (Prentics Hall, 2005).
2. Physics of Atoms and Molecules. B. H. Bransden, A. Bransden, C. J. Joachain (Pearson Education, 2007).

3. Lectures on Quantum Mechanics. A. Das (Hindusthan Book Agency, New Delhi, 2003).

Reference Books:

1. Quantum Mechanics Vol. 1. C. Cohen-Tannoudji, B. Diu, & F. Laloe (Wiley-Interscience publication, 1977).
2. Introduction to Quantum Mechanics. D. J. Griffiths (Pearson Prentics Hall, 2005).

Course: MSMA405-4
Fuzzy Mathematics and Applications-II (Marks - 50)

Total lectures Hours: 50H

Objectives

To present a systematic introduction of the applications of fuzzy mathematics.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

(i) fuzzy logic, fuzzy group theory, fuzzy topology, fuzzy graph theory, fuzzy differential equations.

Skills: The students would be able to

(i) solve fuzzy linear programming and fuzzy multi objective linear programming.

(ii) apply fuzzy graph theory in solving network flow problems.

General competence: The students would gain

(i) the general idea about fuzzy logic, fuzzy group theory, fuzzy topology, fuzzy graph theory, fuzzy differential equations.

Contents:

Fuzzy logic: Fuzzy propositions, fuzzy quantifiers, Fuzzy hedges, Fuzzy implications, Inference from conditional fuzzy propositions. Generalization of hypothetical syllogism, Inference from conditional and qualified propositions. [8H]

Linear Programming Problems with fuzzy resources:

(i) Vendegay's approach

(ii) Werner's approach

L.P.P. with fuzzy resources and objective: Zimmermann's approach.

L.P.P. with fuzzy parameters in the objective function. [10H]

Definition of Fuzzy multiobjective linear programming problems. A brief survey of the methodology of solving fuzzy M.O.L.P. and fuzzy goal programming. [6H]

Fuzzy group theory, ring. Definitions and basic properties. [6H]

Fuzzy topology: definition and basic properties. bitopological spaces. [6H]

Fuzzy differential equations: triangular fuzzy number as coefficients, applications. [6H]

Fuzzy graph theory: fuzzy arcs, paths, cycle. Trees, cut-vertices, fuzzy planar graph. [8H]

Text books:

2) Fuzzy sets and fuzzy logic. G.J. Klir & B Yuan (Prentice Hall of India Ltd. New Delhi 1997).

Reference books

3) Fuzzy Set Theory and its Applications. H. J. Zimmermann (Allied Publishers Ltd. New Delhi 1991).

Course: MSMA406
Project (Marks 50)

Each student is to carry out a project on a topic of his/her own choice in the field of Mathematics and its applications. The nature of the work may be a new investigation or a review of the literature or a detailed analysis of a published research paper or writing an article on a topic at advanced Post Graduate level. A regular student will be guided by a faculty of the Department (Project Supervisor) for carrying out his/her project work. Allocation of Project Supervisors will be decided in the Departmental Committee meeting. Students (other than regular) will carry out project work at their own.

Distribution of marks will be as follows: Written submission: 25 marks, Presentation: 15 marks, Viva-voce: 10 marks

Course: MSMP401
Graph Theory, Set Theory and Logic (Marks-50)

Total Lectures Hours: 50H

Group A
Graph Theory (Marks-25)

Objectives

To understand and apply the fundamental concepts of graph theory based on tools in solving practical problems.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. undirected and directed graphs.
ii. ismorphism of graphs.
iii. Eulerian graphs, Hamiltonian graphs.
iv. various characterizations of trees with applications.
v. bipartite graph and its characterization.
vi. planar and non-planar graphs.
vii. colouring of a graph.
viii. matrix representation of graphs.
ix. applications of graphs in different branches of science and real world problem.

Skills: The students would be able to

i. assimilate various graph theoretic concepts and familiarize with their applications.
ii. efficiency in handling with discrete structures.
iii. efficiency in notions of matrix representation of graph, planarity.
iv. efficiency in solving concrete graph colouring problems.
v. solve real world problems that can be modelled by graphs.

General competence: The students would gain

i. general ideaof graph theory and its real life applications.
ii. understanding about graphic sequence.
iii. experience to apply Euler's formula.
iv. ability to use graphs for various map colouring problems.
v. idea about the application of graphs in computer science.

Contents:

Graph: Undirected graphs and directed graphs with examples and some basic properties, examples and properties of subgraph, isomorphism, walks, paths, cycles, connected components, distance, radius, centre, diameter of a graph, degree sequence, matrix representation of graph, adjacency matrix and incidence matrix of a graph. [8H]
Trees, centres of trees, spanning trees, minimal Spanning tree, Kruskal's algorithm, bipartite graph and its characterization. [7H]
Eulerian Graphs and its characterization, Hamiltonian graphs, Dirac theorem, Ore's theorem. [5H]
Planar and non-planar graphs, statement of Kuratowski theorem and its applications, Euler's formula, five colour theorem, statement of four colour theorem. [5H]

Group B
Set Theory and Logic (Marks – 25)

Objectives: To acquire a systematic knowledge on axiomatic set theory and to familiarize the students on the introduction to basic concepts and techniques of mathematical logic.

Learning outcomes: On completion of the course the students should have the following outcomes defined in terms of knowledge, skill and general competence.

Knowledge: The students will gain knowledge on

i) basic ideas on several axioms like axiom of choice, Zorn's lemma, Zermelo's theorem, Hausdorff maximality principle etc.
ii) the ideas of cardinal numbers and ordinal numbers and also on relevant arithmetic properties
iii) the basic ideas from mathematical logic, namely formal reasoning, for semantics, decidability

Skills: The students will be able to

i) deduce the properties of cardinal numbers and ordinal numbers using addition, multiplication, exponentiation of cardinal numbers
ii) compare the properties of ordinal numbers in respect of commutativity, associativity, distributive property etc. with that of cardinal numbers
iii) corelate the cardinality of countable and uncountable sets
iv) give correct logical arguments
v) find errors in incorrect arguments

General competence: The students will gain

i) overall ideas on axiomatic set theory with several axioms
ii) the ideas on basic properties of cardinal and ordinal numbers.
iii) the ability to discuss logical arguments and their correctness with others
iv) the ability to communicate the basic concepts of logic and their relevance for computer science

Contents:

Set Theory:

Prerequisites: Axiom of choice, Zorn's lemma, Hausdorff's maximality principle, well ordering theorem. [1H]

Cardinal numbers, Schroeder-Bernstein theorem, addition, multiplication and exponentiation of cardinal numbers, the cardinal number $\aleph_0$ and c and their relation. [6H]

Ordinal numbers: Initial segment, orderingof ordinal numbers, transfinite induction, addition and multiplication of ordinal numbers. [5H]

Logic:

Propositional calculus: Premises, conclusion, true-false proposition, validity of arguments, propositional connectives, statement form, truth functions, truth tables. [3H]

Tautologies, contradiction, adequate sets of connectives, an axiomatic system for the propositional calculus. [4H]

Validity rule of conditional proof, formal statement calculus, formal axiomatic theory, deduction theorem and its consequences. [3H]

Quantifiers, free and bound variables, universal and existential, symbolization of languages. [3H]

Text Books:

1. Graph Theory. Nar Sing Deo *(*Prentice-Hall, 1974).
2. Introductory Graph Theory. G. Chartrand (Dover Publications, Inc., 2016).
3. Introduction to Mathematical Logic. E. Mendelson (Taylor& Francis Gr. CRC Press, 2010).
4. Course on Mathematical Logic. S. M. Srivastava (Springer, 2012).

Reference Books

1. Logic for Mathematicians. A. G. Hamilton (Cambridge University Press, 1988).
2. Symbolic Logic,I. M. Copi (Macmillan, New York, 1979).
3. Set Theory and Logic,R. R. Stoll (Dover Publications, Inc. New York, 1963).
4. A First Look at Graph Theory. J. Clark & D. A. Holton (Allied Publishers Ltd., 1995).
5. Introduction to Graph Theory. D. S. Malik, M. K. Sen & S. Ghosh (Cengage Learning Asia, 2014).
6. Introduction to Graph Theory. Douglas Brent West (Prentice Hall, 2001).
7. Graph Theory. Frank Harary (Addison-Wesley, 1971).
8. Graph Theory with Applications. J. A. Bondy & U.S.R. Murty (Macmillan, 1976).

Course: MSMP402

Analysis – II(Marks - 50)

Total lectures Hours: 50

Objectives

To present the fundamental concepts of functional analysis and the idea of integration of a function of several variables.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. isometry, completion of a metric space, uniform boundedness and equi-continuity.
ii. continuity and boundedness of a linear operator, norm of an operator, uniform boundedness, invertible linear operators, Hahn Banach theorem, reflexive spaces.

iii. orthogonal decomposition of Hilbert spaces, Riesz representation theorem, strong and weak convergence of a sequence.
iv. integration over a rectangle, integration over a bounded set, rectifiable sets.
v. improper integral and its properties.
vi. change of variables theorem and its applications.
vii. Green's theorem, divergence theorem, Stoke's theorem.

Skills: The students would be able to

i. find completion of a metric space, determine compact subspaces of C[a,b].
ii. determine existence of inverse of a linear operator, extend a bonded linear operator.
iii. decompose a Hilbert space orthogonally, determine representation of a bounded linear functional over a Hilbert space.
iv. calculate integral over a rectangle, over a bounded set in $\mathbb{R}^n$.
v. evaluate improper integral.
vi. evaluate integration of functions of several variables using change of variables theorem, Green's theorem, divergence theorem, Stoke's theorem.

General competence: The students would gain

i. general idea of functional analysis and the idea of integration of a function of several variables, which will be useful for further studies in Functional Analysis, Multivariate Calculus.
ii. analytical and reasoning skills, which improve their thinking power.

Group-A
Functional Analysis (Marks - 25)

Contents:

Isometry, completion of a metric space, uniformly bounded and equi-continuous functions of C[a,b]with sup norm, compact subspaces of C[a,b], Arzelà Ascoli theorem. [3H]

Linear operators and linear functionals, continuity and boundedness of a linear operator on a normed linear space, norm of an operator, linear operator on finite dimensional normed linear spaces, principle of uniform boundedness and its consequences, invertible linear operators, existence of a bounded inverse linear operator, open mapping theorem. [9H]

Hahn Banach theorem and its applications. [3H]

Conjugate spaces and reflexive spaces with properties, conjugate spaces of C^n, R^n, l_1, l_p $(1< p < \infty)$. [3H]

Minimization of norm problems in inner product spaces, orthogonal decomposition of Hilbert spaces, Riesz representation theorem for bounded linear functionals on Hilbert spaces, Riesz-Fischer theorem. [4H]

Strong and weak convergence of a sequence in a normed linear space, convergence of sequence of bounded linear operators. [3H]

Group B
Calculus of R^n (Marks - 25)

Contents:

The integral over a rectangle, the Riemann condition, set of measure zero in $\mathbb{R}^n$, existence of the integral, evaluation of the integral, Fubini's theorem. [8H]

The integral over a bounded set and properties of the integral, rectifiable sets and their properties. [4H]

Improper integral and its properties, existence of improper integral, evaluation of improper integral. [6H]

Change of variables, change of variables theorem and its applications. [4H]

Statement and applications of Green's theorem, divergence theorem, generalized Stoke's theorem. [3H]

Text Books:

1. Introductory Functional Analysis with Applications. E. Kreyszig (Wiley, 2007).
2. Functional Analysis. G. Bachman and L. Narici (Academic Press, 1966).
3. Elements of Functional Analysis. B.K. Lahiri (The world Press Pvt. Ltd., 1994).
4. Analysis onManifolds. J. R. Munkres (Addison-Wesley Pub. Comp., 1991).

Reference Books:

1. Calculus on Manifolds. M.Spivak (The Benjamin/Cummings Pub.comp.,1965).
2. Introduction to Calculus and Analysis, Vol – II. R. Courant and F. John (Springer, 2004).
3. Basic Multivariate Calculus. T. Marsden (Springer, 2013).
4. Calculus, Vol – II. T. M. Apostol (John Wiley Sons, 1969).
5. A Course in Functional Analysis. J.B. Conway (Springer, 2007).
6. Functional Analysis. A. E. Taylor (John Wiley and Sons, 1958).
7. Functional Analysis. B.V. Limaye (New Age International Publications, 2017).
8. Functional Analysis. J. Muscat (Springer, 2014).
9. Functional Analysis and Applications. A.H. Siddiqi (Springer, 2018).

Course: MSMP403
Topology (Marks – 50)

Total lectures Hours: 50H

Objectives

To present a systematic introduction of the fundamentals course on topology.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. nets and filters as a generalized notion of sequences
ii. some necessary and sufficient conditions of a space to be metrizable or uniformizable and close connection between uniformity and metric.
iii. a brief coverage on compactification especially the Stone-$\check{C}$ech compactification of a space
iv. geometric interpretation of quotient spaces, in particular, identification topology like a piece of wire to circle, a piece of rectangular-shaped paper to cylinder and then to torus etc
v. fundamental groups of a simply connected space and translation of topological problems to algebra and vice-versa.

Skills: The students would be able to

i. manipulate the equivalency between nets and filters
ii. compute uniform completion of a Tychonoff space.
iii. verify whether a space is metrizable.
iv. compute fundamental groups of two topoloigical space and to conclude whether they are homeomorphic.

General competence: The students would gain

i. general idea about the importance nets or filter in the study of topology.
ii. to analyze common features of different concepts of topology like completion of a metrizable or uniform spaces and compactification of a space
iii. expertized in solving many tricky problems in topology by grasping few fundamentals theorems in topology like Nagata-Smirnov metrization theorem and Urysohn's metrization theorem etc.

Contents:

Nets and filter: Directed sets, nets, subnets, filter, subfilter, ultrafilter, universal net, convergence of filter and nets, connections between filter and nets, characterizations of continuity of a function and cluster point of a set in terms of filters and net. [7H]

Quotient spaces: Quotient topology, quotient spaces, canonical decomposition of a continuous mapping, quotient space of subspaces, identification topology. [5H]

Metrizations: Metrizabiltity, operations on metrizable space (topological sum and products), complete metrizable spaces and its properties, metrization theorems (Urysohn's metrization theorem, Nagata-Smirnov metrization theorem). [5H]

Uniform spaces: Uniformities and uniform spaces, neighbourhoods, bases and subbases, uniform isomorphism, relativization, products, Cauchy net and filter, complete uniform spaces, completion, totally bounded and uniqueness of uniformity on compact spaces, the gauge of uniformity. [10H]

Compactifications: One-point compactification, Stone-$\check{C}$ech compactification and its characterization in terms of extension property. [5H]

Paracompact spaces: Definition, examples, properties, partition of unity, characterizations of paracompactness, closed hereditary and closed under topological sum, invariant under closed mapping (Michael theorem) and perfect mapping, Tamano theorem. [10H]
Algebraic topology: Homotopy of paths, covering spaces, fundamental group, fundamental group of the circle. [8H]

Text Books:

1. General Topology. R. Engelking, 2nd edition (Helderman Verlag, 1989)
2. Topology. J. Muknkres, 2nd edition (Pearson, 2015)
3. Introduction to General Topology. K. D. Joshi, 2nd edition (New Age International, 2018)
4. Introduction to General Topology. J.L. Kelly, 1st edition (Springer India, 1988)

Reference Books:

1. Topology. J. Dugundji, 1st edition (Ubs Publishers Distributors Ltd, 1999).
2. Counter examples in topology. L.A. Steen, J.A.Seebach, 2nd edition (Dover Publication,Inc.,1995)
3. General Topology. S. Willard (Dover Publication,INC, 2004)
4. Encyclopedia of General Topology. P.K.Hart,J. Nagata, J.E. Vaughan, 1st edition (Elsevier,2003)
5. Introduction to General Topology. S.T. Hu, 1st edition (Holden Day, 1966)
6. Foundation of Topology. C.W. Patty, 2nd edition (Jones and Bartlett Publishers, Inc, 2008)
7. General Topology (Chapter 5-10). N. Bourbaki, 1st edition (Springer, 1998).

Course: MSMP404-1
Advanced Functional Analysis-II (Marks-50)
Total Lectures Hours: 50H

Objective

The course is designed in such a way that student can learn fundamental topics of one of the core areas of functional analysis namely theory of approximations and theory of Banach Algebra and the like. The course is framed with a view to learn certain topological-algebraical structures on some normed spaces and the methods by which the knowledge of these methods can be applied to operator theory. The objectives of the course are to learn further some important results of Banach Algebra together with the study of spectral theory the elements with its applications.

Learning outcomes

Knowledge:

The students learn

- the basic ideas on theory of approximations
- representation theory of some normed spaces

- the basic results on Algebra together with
  - (i) Banach Algebra
  - (ii) division algebra
  - (iii) spectral theory of elements
  - (iv) quotient algebra
  - (v) Banach * algebara
  - (vi) theory of involutions
  - (vii) B* algebra

Skills:

- Students who has had a course of advanced calculus and a minimum of algebra and topology finds no difficulty on this course
- The students are motivated by experiencing their knowledge on such area that facilitates them to work out various problems on the allied areas of functional analysis.

General Competence:

(iii) It helps the students to read and to learn evaluate critically further topics in functional analysis

It motivates the students to make them easier in exercising the applications of such coursein higher studies of mathematical sciences and mathematical physics.

Contents:

Weierstrass approximation theorem, best approximation theory, uniqueness criterion for best approximation. [6H]

Algebra, sub-algebra, Stone Weierstrass theorem. [6H]

Banach Algebra, invertible and non-invertible elements, resolvent set, resolvent function, spectrum, compactness of spectrum, spectral radius, spectral mapping theorem for polynomials, spectral radius formula, division algebra, Gelfand-Mazur theorem, topological divisors of zero. [12H]

Homomorphism and quotient algebra, ideals, maximal ideals, Gelfand mapping, Banach* algebra, B* algebra, radical, involution, *isomorphism, centralizer, positive functional. [8H]

Gelfand topology, Gelfand-Neumark theorem. [6H]

Weak topology, weak* topology, Banach Alaoglü theorem. [6H]

Representation theorems for C_0, C[a,b](with sup-norm) and $L_p[a,b](1 \le p < \infty)$. [6H]

Text Books:

1. Elements of Functional Analysis. I.J.Madox (Universal Book Stall,1992)
2. Introductory Functional Analysis with Applications. E. Kreyszig (Wiley Eastern,1989).
3. Functional Analysis. W. Rudin (TMG Publishing Co. Ltd., New Delhi,1973).
4. Banach Algebras: An Introduction. R. Larsen (Mercel Dekker, Inc., 1973).

Reference Books:

1. Functional Analysis. Joseph Muscat (Springer, 2014).
2. General Theory of Banach Algebras. C. E. Rickart,(Robert E. Krieger Publishing Cc., Inc., 1974)
3. Banach Algebras and the General Theory of *Algebras (Vol-I,II). T. W. Palmer (Cambridge University Press, 1994)
4. A Course in Commutative Banach Algebra. E. Kenneth (Springer,2009).

Course: MSMP404-2

Advanced Differential Geometry-II (Marks-50)

Total Lectures Hours: 50H

Objectives

To present systematically some structures on manifolds based on the curvature tensor, Einstein field equations, complex manifolds, contact metric manifolds and their submanifolds.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. some structures on manifolds like recurrent, pseudo symmetric manifolds etc.
ii. Einstein field equations.
iii. almost complex manifolds.
iv. submanifolds of complex manifolds.
v. contact metric manifolds.
vi. Sasakian manifolds.
vii. K-contact manifolds.
viii. submanifolds of various contact metric manifolds.

Skills: The students would be able to

i. apply the notion of Cartan's symmetric manifolds and its generalizations.
ii. apply the concept of contact manifolds in physics.
iii. develop some complex and contact metric structures.
iv. explore many fascinating aspects of relativity, such as the behaviour of Einstein field equation, the Schwarchild spacetimes, the Robertson-Walker spacetimes.

General competence: The students would gain

i. general idea of Cartan's symmetric manifolds and its generalizations which will be useful for further studies.
ii. understanding about complex manifolds and contact metric manifolds.
iii. experience to construct submanifolds of complex and contact metric manifolds using mathematical tools.
iv. about applications on contact metric and complex manifolds.

Contents:

Structures on Manifolds: Cartan's symmetric manifolds, recurrent manifolds, semi-symmetric manifolds, pseudosymmetric manifolds, Einstein field equation, Schwarchild spacetimes, Robertson-Walker spacetimes. [15H]

Complex Structures:Almost complex manifolds, Nijenhuis tensor, contravariant and covariant almost analytic vector fields, almost Hermite manifolds, Kähler manifolds, almost Tachibana manifolds, holomorphic sectional curvature, submanifolds of complex manifolds. [15H]

Contact Structures:Contact manifolds, K-contact manifolds, Sasakian manifolds, Kenmotsu manifolds, Trans-Sasakian manifolds, cosymplectic manifolds and their submanifolds. [20H]

Text Books:

1. Contact Manifolds in Riemannian Geometry. D. E. Blair (Birkhausher, 2005).
2. Complex and Contact manifolds. U. C. De & A. A. Shaikh (Narosa Publ. Pvt. Ltd, New Delhi, 2009).
3. Geometry of Submanifolds. B. Y. Chen (Dover Publications Inc., 2019).

Reference Books:

1. Structure on Manifolds. K. Yano & M. Kon (World Scientific, 1984).
2. Semi-Riemannian Geometry with Application to Relativity. B. O'Neill (Academic Press, 1983).
3. An introduction to contact topology. H. Geiges (Cambridge Univ. Press, 2008).
4. Contact geometry and non-linear differential equations. A. Kushner, V. Lychagin & V. Rubtsov (Cambridge Univ. Press, 2007).

Course: MSMP404-3

Advanced Complex Analysis-II (Marks-50)

Total Lectures Hours: 50H

Objectives

To present the theory of meromorphic functions with a focus on values distributions of meromorphic functions and its applications.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

i. On harmonic functions and their characterization.
ii. On Doubly periodic functions, Weierstrass elliptic function p(z) and their basic properties
iii. On Nevalinna theory of value distributions.
iv. On further applications in various branches of mathematical sciences.

Contents:

Harmonic functions, characterization of harmonic functions by mean- value property, Poisson's integral formula, Dirichlet problem for a disc. [12H]
Doubly periodic functions, Weierstrass elliptic function p(z), addition theorem for p(z), differential equation satisfied by p(z), the numbers e_1, e_2, e_3. [14H]
Meromorphic functions, definitions of the functions N(r,a), m(r,a) and T(r,f), Nevanlinna's first fundamental theorem, Cartan's identity and convexity theorems, orders of growth, order of meromorphic function, comparative growth of log M(r) and T(r), Nevanlinna's second fundamental theorem, estimation of S(r) (statement only), Nevanlinna's theorem on defiant functions, Nevanlinna's five-point uniqueness theorem, Milloux theorem. [24H]

Text Book:

1. Theory of Functions of a Complex Variables Vol. I & II.A. I. Markusevich (Printice-Hall, 1965).
2. Meromorphic functions. W. K. Hayman (Oxford University Press, 1964).
3. Complex Analysis.L. V. Ahlfors, 3rd edition (McGraw-Hill, 1979).

Reference Books:

1. Theory of Functions,E. C. Titchmarsh, 2nd edition(Oxford University Press, 1970).
2. Introduction to the Theory of Function of a Complex Variable. E. T. Copson (Oxford University press, 1970).
3. Theory of Analytic Functions,H. Cartan (Dover Publication, 1995).
4. Elements of The Theory of Elliptic and Associated Functions with Applications.M. Dutta and L. Debnath (World Press Pvt., 1965).
5. L. Yang, *Value distribution theory*, Springer-Verlag Berlin Heidelberg, 1993.

Course: MSMP404-4

Measure and Integration-II (Marks – 50)

Total Lectures Hours: 50H

Objectives

To present the concepts of signed measure, complex measure, product measure and $L_p$spaces.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. signed measures, complex measure, integrability of functions with respect to signed measure and complex measure.
ii. product measures and their properties.
iii. $L_p(\mu)$ and $L_p[a,b]$ spaces ($1 \le p \le \infty$),dense subspaces of $L_p[a, b]$ spaces.

Skills: The students would be able to

i. decompose a measurable space with respect to a signed measure, decompose a signed measure into mutually singular measures.
ii. Integrate measurable functions with respect to signed measure, complex measure and product measure.
iii. determine dense subspaces of $L_p[a, b]$ spaces.

General competence: The students would gain

i. general idea of signed measure, complex measure, product measure and L_p spaces, which will be useful for further studies in real analysis, probability theory.
ii. analytical and reasoning skills, which improve their thinking power.

Contents:

Signed measures, positive set, negative set, mutually singular measures, Hahn decomposition theorem, Jordan decomposition theorem, Radon-Nikodym theorem, Radon-Nikodym derivative, Lebesgue decomposition theorem, complex measure, integrability of functionsw.r.t. signed measure and complex measure, total variation of a complex measure. [25H]

Measurable rectangles, elementary sets, measurable sections, product measures and their properties, Fubini's theorem. [10H]

$L_p(\mu)$ and $L_p[a,b]$ spaces ($1 \le p \le \infty$),Hölder and Minkowski inequalities, completeness and other properties of $L_p[a, b]$ spaces, dense subspaces of $L_p[a, b]$ spaces, bounded linear functionals on $L_p[a, b]$ spaces and their representations. [15H]

Text Books:

1. Measure Theory. P. R. Halmos (Springer-Verlag, 1974).
2. Measure and Integration. S. K. Berberian (The Macmillan Co., 1965)
3. An Introduction to Measure and Integration.I. K. Rana (Narosa Publishing House, 1997).

Reference Books:

1. Measure Theory and Integration. G. D. Barra (New Age International (P) Ltd, 2013).
2. Real and Abstract Analysis.E. Hewitt and K. Stormberg (John Wiley, 1965).
3. Real Analysis. H.L.Royden, 3rd edition (PHI, 2002).
4. Theory of Functions of a Real Variable, Vol. I & II. I. P. Natanson (Fedrick Unger Publi. Co., 1961).

5. Real and Complex Analysis. W. Rudin (Tata Mc Graw Hill, 1993).
6. Measure, Integration and Function Spaces.Charles Swartz (World Scientific, 1994).

Course: MSMP405-1

Euclidean and non – Euclidean Geometries-II (Marks-50)

Total Lectures Hours: 50H

Objectives

To present a systematic study and development of the discovery of non-Euclidean geometries, hyperbolic plane geometry, spherical plane geometry and Riemannian geometry.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. history and subsequent developments of non-Euclidean geometries.
ii. classification of parallels.
iii. some models of non-Euclidean geometries.
iv. geometric transformations on hyperbolic plane.
v. hyperbolic plane geometry.
vi. spherical plane geometry.
vii. Riemannian geometry.
viii. acquire the concept of infinitely many parallels of a line in hyperbolic geometry.
ix. visualize hyperbolic surfaces in nature and different branches of science.
x. conformal, concircular and projective transformations on a Riemannian space.

Skills: The students would be able to

i. determine geometric transformations on hyperbolic plane.
ii. apply group of automorphisms to geometric problems.
iii. find length minimizing curves in $H^2$and to determine isometries of H^2.
iv. compute formula for angle of parallelism in the Poincaré disk model.
v. compute the distance between two points on elliptical plane and hyperbolic plane.
vi. application of spherical geometry in astronomy and to determine the position of a point on earth surface.

General competence: The students would gain

i. expertise the growth and development from Euclidean to non-Euclidean geometries.
ii. general idea of hyperbolic and spherical geometry which will be useful for further advanced studies.

iii. understanding about Riemannian geometry.
iv. experience to construct Riemannian, Lorentzian and semi-Riemannian metrics.
v. idea of the differences among the formulae of finding length of curves in Euclidean, spherical and hyperbolic planes.

Contents:

The discovery of non-Euclidean geometry: History and subsequent developments of non-Euclidean geometry, non-Euclidean Hilbert planes, universal non-Euclidean theorem, parallels admitting a common perpendicular, limiting parallel rays, hyperbolic planes, classification of parallels, perpendicular bisector theorem (statement only). [5H]
Models of non-Euclidean geometry: Consistency of hyperbolic geometry, Beltrami–Klein model, Poincaré models, isomorphism of Beltrami-Klein and Poincaré models, inversion in circles, Poincaré length, formula for angle of parallelism in the Poincaré disk model, congruence, perpendicularity in the Beltrami–Klein model, projective nature of the Beltrami–Klein model. [7H]
Geometric Transformations on hyperbolic plane: Klein's Erlanger programme, group of automorphisms, applications to geometric problems, motions and similarities, reflections, rotations, translations, half-turns. [5H]
Hyperbolic plane geometry: Poincaréupper half-plane, line elements on open subsets of R^2, Poincaré metrics on H^2, length minimizing curves in H^2, distance function on H^2, triangles in H^2, two-point homogeneity of H^2, isometries of H^2. [14H]
Spherical Plane Geometry: Definition and examples, the tangent plane at any point of the sphere S^2, triangles in S^2, action of SO(3) on S^2, SO(3) preserves length, isometries of S^2, Geometric version of Euler's theorem, two-point homogeneity of S^2. [14H]
Riemannian geometry: Riemannian space, Riemannian, Lorentzian and semi-Riemannian metrics, Riemannian curvature, conformal, concircular and projective transformations on a Riemannian space and their invariants (statements only). [5H]

Text Books:

1. Euclidean and non-Euclidean Geometries: Development and History. Marvin Jay Grenberg, 4th edition (W. H. Freeman and Company, New York, 2008).
2. An Expedition to Geometry. S. Kumaresan & G. Santhanam (Hindustan Book Agency, 2005).

Reference Books:

1. What is mathematics? R. Courant & H. Robbins (Oxford Univ. Press, New York, 1941).
2. A history of mathematics. C. B. Boyer & U. Merzbach, 2nd edition (New York, Wiley, 1991).
3. Introduction to geometry. H. S. M. Coxeter, 2nd edition (New York, Wiley, 2001).
4. Foundations of hyperbolic manifolds. J. G. Ratcliffe, 2nd edition (New York, Springer, 2006).

5. Introduction to non-Euclidean geometry. H. E. Wolfe (New York, Holt, Rinehart and Winston, 1945).

Course: MSMP405-2

Commutative Algebra - II(Marks-50)

Total Lectures Hours: 50H

Objectives

To present a systematic introduction of the fundamentals course on commutative algebra II.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. valuation ring, Dedeking domain and Ramification formula.
ii. filtered ring and module, graded ring and graded R-module.
iii. Complexes projective resolution of R-module, Ext and Tor functors.
iv. Krull dimension, height and co-height prime ideal.

Skills: The students would be able to

i. Work on homomorphism of filtered module, graded ring etc.
ii. compute projective dimension and global dimension of a ring.
iii. height and co-height of prime ideal.

General competence: The students would gain

i. General idea about the importance Artin-Rees lemma, inverse system of R-module.
ii. To analyze homological theory and homological dimension
iii. Expertized in solving many tricky problems in commutative algebra

Contents:

(Throughout R is assumed to be a commutative ring with 1)

Valuation ring: Valuation ring-examples and properties, discrete valuation ring-examples and properties, Dedekind domain-examples and properties, Ramification formula. [8H]

Completions: Filtered rings and module, homomorphism of filtered module, graded ring, fraded R-module, homomorphism of graded modules, completion, I- filtration, I-stable filtration, Artin-Rees lemma, inverse system of R-module, I-adic filtration, I-adic topology, I-adic completion, Krull's intersection theorem, associated graded ring, Hensel's lemma, implicit function theorem. [12H]

Homology: Complexes, homotopically equivalent complexes, acyclic complex, derived functors, projective resolution of R-module, projective resolution of exact sequence of R-module, projective dimension, global dimension of a ring, injective R-module, any R-module can be embedded in an injective R-module, $Hom_R(\_\_, N), Ext\ and\ Tor$ functors, injective dimension, local criterion of flatness, homological dimension. [12H]
Dimension: Hilbert-Samuel polynomials, Krull dimension, height and co-height of a prime ideal, dimension theorem, Krull's principal ideal theorem, dimension of algebras, depth, Cohen-Macaulay models, Cohen-Macaulay (C.M.) ring, Macaulay's theorem. [12H]

Regular local ring: Regular local ring, homological characterization, normality condition, complete local ring. [6H]

Text books:

1. Commutative Algebra. N.S. Gopalakrishnan, 2nd edition (Orient Blackswan Private Limited, 2017).

Reference books:

1. Introduction to Commutative Algebra. M. Atiya and I.G. MacDonald, 1st edition (Levant Books, India, 2007).
2. Abstract Algebra. D.S. Dummit & R.M. Foote, 3rd edition (Wiley, 2016).
3. Module Theory: An Approach to Linear Algebra. T. S. Blyth, 1st edition(Clarendon Press, 1977).
4. Basic Algebra II. N. Jacobson, 1st edition (Hindustan Publishing Corporation, India, 1984).
5. Commutative Algebra-with a View Toward Algebraic Geometry. D. Eisenbud, 1st edition (Springer-Verlag New York, 1995).

Course: MSMP405-3
Advanced Operator Theory -II (Marks-50)

Total Lectures Hours: 50H

Objectives: To present a systematic knowledge on operator theory in advanced level focusing mainly to spectral representation theorems of different kinds of operators.

Learning outcomes: On completion of the course the students should have the following outcomes defined in terms of knowledge, skill and general competence.

Knowledge: The students will acquire knowledge on

i. projection operators, positive operators, spectral family, unbounded operators and spectral representation theorems of different kinds of operators like compact normal, bounded self-adjoint operators, unitary operators
ii. various properties of orthogonal projections, positive operators and square root of positive operators

iii. spectral properties of compact normal operators, self adjoint operators, unbounded linear operators like multiplication operator and differential operator
iv. spectral representation theorem of compact normal operators in terms of orthogonal projections
v. spectral representation theorem of bounded self adjoint operator and unitary operators in terms of spectral family

Skills: The students will be able to

i. find spectrum, eigen values of various operators like compact normal operators, self adjoint operators etc.
ii. solve various problems related to spectral properties
iii. represent an operator defined on a specific domain in terms of orthogonal projections
iv. to compare the main features of spectral representations of different kinds of operators

General competence: The students will gain

i. over all ideas on spectral theory of operators like bounded normal, compact, compact normal, bounded self adjoint, unbounded self adjoint etc.
ii. the ideas of spectral representation of some operators like bounded self adjoint and unitary operators as an Riemann Stieltjes integral in terms of spectral family.
iii. overall ideas on operator theory and its application.

Contents:
Infinite orthogonal direct sums, commutatively convergence of infinite series of operators, spectral theorem of compact normal operators. [6H]
Spectral theory of bounded self adjoint operators, resolvent set, eigen values, spectrum of bounded self adjoint operators. [6H]

Projection operators on Hilbert spaces, product of projection operators, partial ordering of projection operators, sum and difference of projection operators, monotonically increasing sequence of projection operators, positive operators, product of positive operators, monotone sequences of positive operators, square root of positive operators. [14H]

Spectral family(decomposition of unity) of an interval [a,b], spectral family of bounded self adjoint linear operators, spectral family associated with an operator, spectral representation theorem(representation as Stieltjes integral) of bounded self adjoint operators. [8H]
Unbounded linear operators on Hilbert spaces: Hellinger-Toeplitz theorem, spectral properties of arbitrary self-adjoint operators, Wecken's lemma, spectral theorem of unitary operators, Cayley transform of self adjoint operators, spectral theorem of arbitrary self-adjoint operators. [12H]

Multiplication operator and differentiation operator, self-adjointness and unboundedness property of multiplication operators and differentiation operators, spectral properties of multiplication operators. [4H]

Text books:

1. Functional Analysis. G. Bachman and L. Narici (Academic Press, 1966).
2. Introductory Functional Analysis with Applications. E.Kreyszig (Wiley, 1989).

Reference books:

1. Elements of Functional Analysis. B. K. Lahiri (The World Press Pvt. Ltd., Kolkata, 1994).
2. Functional Analysis. B. V. Limaye (Wiley Eastern Ltd, New Delhi, 1981).
3. Introduction to Topology and Modern Analysis. G. F. Simmons (McGraw-Hill, New York, 1958).
4. Functional Analysis. K. Yosida (Springer Verlag, New York, 3rd Edn, 1990).

Course: MSMP405-4

Ergodic Theory - II (Marks - 50)

Total Lectures Hours: 50H

Objectives

To present a systematic introduction of the fundamentals course on ergodic theory II.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. Topological dynamics and application of ergodicity over it.
ii. Nice behaviour of measure on metric space.
iii. Relation between topological entropy and measure theoretic entropy.
iv. Variation principal related to topological pressure.

Skills: The students would be able to

i. To compute various problems in topological dynamics using measure preserving transformations.
ii. Classify of measure preserving transformation upto conjugacy.
iii. Interpret ergodicity and mixing.
iv. To calculate topological entropy.

General competence: The students would gain

i. General idea about the importance ergodic theory
ii. To analyze common features of different concepts in ergodic theory

iii. Expertized in solving many tricky problems in ergodic theory.

Contents:

Topological dynamics, examples, minimality, the non-wandering set, topological transitivity, topological conjugacy and discrete spectrum, expansive homeomorphism. [10H]

Measures on metric space, invariant measure for continuous transformations, interpretations of ergodicity and mixing, relation of invariant measure to non-wandering sets, periodic points and topological transitivity, unique ergodicity, examples. [10H]
Topological entropy, definitions using open covers, Bowen's definition, calculation of topological entropy. [6H]
Relation between topological entropy and measure-theoretic entropy,entropy map, the variational principle, measure with maximal entropy, entropy of affine transformations,distribution of periodic points,definition of measure-theoretic entropy using the metrics d_n. [12H]
Topological pressure, properties of pressure, the variational principle, pressure determines$M(X, T)$, equilibrium states. [12H]

Text books:

1. An Introduction to Ergodic Theory. P. Walters, 1st edition (Springer, 2004).

Reference books:

1. Ergodic theory and information. Patrick Billingsley, 1st edition (Robert E. Krieger Publishing Co., 1978).
2. Basic Ergodic theory. M. G. Nadkarni, TRIM 6 (Hindustan Book Agency, 1995).
3. Recurrence in ergodic theory and combinatorial number theory. H. Furstenberg, 1st edition (Princeton University Press 1981).
4. Ergodic Theory. K. Petersen, Cambridge Studies in Advanced Mathematics (2) (Cambridge University Press, 1989).
5. Ergodic theory with a view towards number theory. M. Einsiedler and T. Ward, 1st edition (Springer-Verlag, London, 2011).

MSMP405-5

Algebraic Topology-II (Marks - 50)

Total lectures Hours:50H

Objectives

To present a systematic introduction of the fundamentals course on algebraic topology II.

Learning outcomes

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. the basic concepts singular homology and cohomology group by Eilenberg and steenrod axioms.
ii. importance of singular cohomology ring.
iii. fibre bundles and fibre products.

Skills: The students would be able to

i. calculate homology and cohomology groups for various spaces like circle, projective spaces etc.
ii. compute cohomology ring for projective spaces.
iii. investigate induce bundles, vector bundles and their morphisms.
iv. analyze homology exact sequence of a fibre bundle.

General competence: The students would gain

i. general idea about the importance algebraic topology
ii. to analyze relation between $H_1(X)$ and $\pi_1(X)$
iii. expertize in solving many tricky problems in algebraic topology.

Contents:

Introduction of singular homology and cohomology group by Eilenberg and steenrod axioms, existence and uniqueness of singular homology and cohomology theory. [14H]
Calculation of homology and cohomology groups for circle, projective spaces, torus relation between $H_1(X)$ and $\pi_1(X)$. [14H]
Singular cohomology ring, calculation of cohomology ring for projective spaces.
Fibre bundles: Definitions and examples of bundles and vector bundles and their morphisms, cross sections, fibre products, induced bundles and vector bundles and their morphisms, cross sections, fibre products, induced bundles and vector bundles, homotopy properties of vector bundles, homology exact sequence of a fibre bundle. [22H]

Text Books:

1. Algebraic Topology.A. Hatcher, 1st edition (Cambridge University Press, 2003).
2. Algebraic Topology. J. R. Munkres, 1st edition (Westview Press, 1996).

Reference Books:

1. Basic Concepts of Algebraic Topology. F. H. Croom, 1st edition (Springer, NY, 1978.)
2. Algebraic Topology. E. H. Spanier, 1st edition (McGraw-Hill,1966)

3. A Basic Course in Algebraic Topology. W. S. Massey (Springer-Verlag, New York Inc., 1991).
4. Lecture Notes on Elementary Topology and Geometry. I. M. Singer & J. A. Thorpe, 1st edition(Springer, India 2003).

Course: MSMP406
Project (Marks 50)

Each student is to carry out a project on a topic of his/her own choice in the field of Mathematics and its applications. The nature of the work may be a new investigation or a review of the literature or a detailed analysis of a published research paper or writing an article on a topic at advanced Post Graduate level. A regular student will be guided by a faculty of the Department (Project Supervisor) for carrying out his/her project work. Allocation of Project Supervisors will be decided in the Departmental Committee meeting. Students (other than regular) will carry out project work at their own.

Distribution of marks will be as follows: Written submission: 25 marks, Presentation: 15 marks, Viva-voce: 10 marks